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Rigid and Control Modes Aerodynamic Unsteady Forces Aeroservoelastic Modeling

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An important aspect of aeroservoelasticity is the analysis of interactions between rigid, elastic, and control modes. The stability and control derivatives were determined for a high number (90) of flight test conditions expressed in terms of Mach number, altitudes, and angles of attack. In this paper, the rigid and control mode aerodynamic forces for an F/A-18 aircraft are calculated and validated by two methods (numerical and analytical). The results obtained here were found to be identical for all flight test conditions.

Nomenclature

b	=	wing span
C_D	=	drag coefficient
C_L	=	roll moment coefficient
C_{Lift}	=	lift coefficient
C_M	=	pitch moment coefficient
C_N	=	yaw moment coefficient
C_Y	=	side force coefficient
$\bar{c}$	=	airfoil chord
H	=	altitude
H_{xa}	=	angular moment on the x_a axis
H_{ya}	=	angular moment on the y_a axis
H_{za}	=	angular moment on the z_a axis
L	=	linear moment along the x_a axis
M	=	linear moment along the y_a axis
m	=	mass
N	=	linear moment along the z_a axis
p	=	roll angular speed
q	=	pitch angular speed
q_{dyn}	=	dynamic pressure
r	=	yaw angular surface speed
S	=	wing surface
u	=	linear speed on the x_a aircraft system of coordinates axis
V	=	true airspeed
v	=	linear speed on the y_a aircraft system of coordinates axis
w	=	linear speed on the z_a aircraft system of coordinates
α	=	angle of attack
β	=	sideslip angle
$\delta_{\mathrm{lat}_{\mathrm{AIL}}}$	=	aileron angle for lateral motion
$\delta_{\mathrm{lat}_{\mathrm{HT}}}$	=	horizontal tail angle for lateral motion
$\delta_{\mathrm{lat}_{\mathrm{LEF}}}$	=	leading-edge flaps for lateral motion
$\delta_{\mathrm{lat}_{\mathrm{TEF}}}$	=	trailing-edge flaps for lateral motion
$\delta_{\mathrm{lat}_{\mathrm{RUD}}}$	=	rudder angle for lateral motion
$\delta_{\mathrm{long}_{\mathrm{AIL}}}$	=	aileron angle for longitudinal motion
$\delta_{\mathrm{long}_{\mathrm{HT}}}$	=	horizontal tail angle for longitudinal motion
$\delta_{\mathrm{long}_{\mathrm{LEF}}}$	=	leading-edge flaps for longitudinal motion
$\delta_{\mathrm{long}_{\mathrm{TEF}}}$	=	trailing-edge flaps for longitudinal motion
$\delta_{\mathrm{long}_{\mathrm{RUD}}}$	=	rudder angle for longitudinal motion
θ	=	pitch angle
ρ	=	air density
ϕ	=	roll angle
ψ	=	yaw angle

I. Introduction

AEROSERVOELASTICITY studies are very important in the aircraft industry. Following an extensive bibliographic research in the field, we were not able to find a well-documented formulation for rigid and control mode aerodynamic forces for aeroservoelasticity; therefore, we formulated and validated a novel method in this paper. Our new formulation will calculate and validate the aerodynamic rigid and control modes forces on an F/A-18 aircraft by use of flight dynamics rigid body theory instead of by using the doublet lattice method (subsonic regime) and the constant pressure method (supersonic regime) employed in finite element codes such as STARS [1] or Nastran [2].

If the rigid aerodynamics are calculated with aeroelasticity code by aeroelasticity specialists, however, for aeroservoelasticity calculations, there is the need to interact with flight dynamics specialists to obtain from them the stability and control derivatives. Both specialists need to interact with each other for aeroservoelasticity studies. For this reason, there is a need for the common formulation here presented (interaction between aeroelasticity with flight dynamics and control). There is therefore the need of interaction studies between these specialists to obtain the rigid and control modes aerodynamics studies by use of stability and control derivatives for aeroservoelasticity studies.

The aerodynamic forces for the rigid-to-rigid mode interactions Q_{rr} (dimensions 6×6) and for the rigid-to-control mode interactions Q_{rc} (dimensions 6×10), calculated with finite element code Stars, will be replaced by Q_{rr} and Q_{rc} values calculated with the two schemes, analytical and numerical, presented in this paper.

The analytical formulation is presented in Secs. II, III, and IV, whereas the numerical formulation is presented in Sec. V. Details of the first simulation scheme with stability derivatives in the wind system of coordinates are explained in Sec. II, whereas Sec. III presents the first scheme with state variables introduced in the aircraft closed loop. Section IV presents the analytical formulations for the aerodynamic forces for rigid-to-rigid and rigid-to-control interaction mode calculations. Results are validated by two formulations for 90 flight test conditions.

II. First Simulation Scheme with Stability Derivatives in the Wind System of Coordinates

The detailed first scheme is shown in Fig. 1. This scheme can be written in its equivalent form, shown in Fig. 2. The details of blocks 1–9 are presented next.

Block 1 description: Two sets of stability and control derivatives in the wind coordinate system are known, provided by NASA Dryden Flight Research Center (DFRC), for various flight conditions

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characterized by Mach numbers, altitudes, and angles of attack. We express the aircraft behavior with the second state-space matrix equation:

$$y = A_{\mathrm{NASA}}x + B_{\mathrm{NASA}}u = [A_{\mathrm{NASA}} \quad B_{\mathrm{NASA}}]\begin{bmatrix} x \\ u \end{bmatrix} = [A_t]\begin{bmatrix} x \\ u \end{bmatrix} \quad (1)$$

Where the A_t matrix has the dimensions (6×19), and A_{NASA} is the stability derivative coefficients matrix of dimensions (6×9), in which rows 1–6 correspond to the derivatives of C_L, C_M, C_N, C_D, C_{Lift}, and C_Y with respect to the variables in columns 1–9 which are $q, \alpha, V, \theta, H, p, r, \beta$, and ϕ. The control derivative coefficients matrix B_{NASA} has the dimensions (6×10) and is divided into two matrices corresponding to the longitudinal and lateral aircraft motion, which are denoted by $B_{\mathrm{long}_{\mathrm{NASA}}}$ and $B_{\mathrm{lat}_{\mathrm{NASA}}}$. Therefore, B_{NASA} is written as $B_{\mathrm{NASA}} = [B_{\mathrm{long}_{\mathrm{NASA}}} \quad B_{\mathrm{lat}_{\mathrm{NASA}}}]$ where $B_{\mathrm{long}_{\mathrm{NASA}}}$ is a (6×6) matrix containing the derivatives of the same coefficients as the ones of A_{NASA} (which are C_L, C_M, C_N, C_D, C_{Lift}, C_Y) with respect to the longitudinal control surfaces $\delta_{\mathrm{long}_{\mathrm{AIL}}}$, $\delta_{\mathrm{long}_{\mathrm{HT}}}$, $\delta_{\mathrm{long}_{\mathrm{RUD}}}$, $\delta_{\mathrm{long}_{\mathrm{LEF}}}$, and $\delta_{\mathrm{long}_{\mathrm{TEF}}}$; $B_{\mathrm{lat}_{\mathrm{NASA}}}$ is a (6×6) matrix containing the derivatives of the same coefficients as the ones of A_{NASA} (which are C_L, C_M, C_N, C_D, C_{Lift}, C_Y) with respect to the lateral control surfaces $\delta_{\mathrm{lat}_{\mathrm{AIL}}}$, $\delta_{\mathrm{lat}_{\mathrm{HT}}}$, $\delta_{\mathrm{lat}_{\mathrm{RUD}}}$, $\delta_{\mathrm{lat}_{\mathrm{LEF}}}$, and δ_{lat}. The output, state, and input vectors are given by the following equations:

$$y = \begin{pmatrix} \Delta C_L & \Delta C_M & \Delta C_N & \Delta C_D & \Delta C_{\mathrm{Lift}} & \Delta C_Y \end{pmatrix}^T \quad (2)$$

$$x = \begin{pmatrix} \Delta q & \Delta\alpha & \Delta V & \Delta\theta & \Delta H & \Delta p & \Delta r & \Delta\beta & \Delta\phi \end{pmatrix}^T \quad (3)$$

$$u = [\Delta\delta_{\mathrm{long}_{\mathrm{LEF}}} \quad \Delta\delta_{\mathrm{long}_{\mathrm{TEF}}} \quad \Delta\delta_{\mathrm{long}_{\mathrm{AIL}}} \quad \Delta\delta_{\mathrm{long}_{\mathrm{HT}}} \quad \Delta\delta_{\mathrm{long}_{\mathrm{RUD}}} \quad \ldots \quad \Delta_{\mathrm{lat}_{\mathrm{LEF}}} \quad \Delta_{\mathrm{lat}_{\mathrm{TEF}}} \quad \Delta_{\mathrm{lat}_{\mathrm{AIL}}} \quad \Delta_{\mathrm{lat}_{\mathrm{HT}}} \quad \Delta_{\mathrm{lat}_{\mathrm{RUD}}}]^T \quad (4)$$

The Taylor series approximations of stability and control derivatives at the trim position $\Delta C_L, \Delta C_M, \Delta C_N, \Delta C_D, \Delta C_{\mathrm{Lift}}$, and ΔC_Y can be written as follows:

$$\begin{aligned} \Delta C_L = {} & \frac{\partial C_L}{\partial q}\Delta q + \frac{\partial C_L}{\partial \alpha}\Delta\alpha + \frac{\partial C_L}{\partial V}\Delta V + \frac{\partial C_L}{\partial \theta}\Delta\theta + \frac{\partial C_L}{\partial H}\Delta H \\ & + \frac{\partial C_L}{\partial p}\Delta p + \frac{\partial C_L}{\partial r}\Delta r + \frac{\partial C_L}{\partial \beta}\Delta\beta + \frac{\partial C_L}{\partial \phi}\Delta\phi \\ & + \frac{\partial C_L}{\partial \delta_{\mathrm{Pi}_{\mathrm{AIL}}}}\Delta\delta_{\mathrm{Pi}_{\mathrm{AIL}}} + \frac{\partial C_L}{\partial \delta_{\mathrm{long}_{\mathrm{HT}}}}\Delta\delta_{\mathrm{long}_{\mathrm{HT}}} \\ & + \frac{\partial C_L}{\partial \delta_{\mathrm{long}_{\mathrm{RUD}}}}\Delta\delta_{\mathrm{long}_{\mathrm{RUD}}} + \frac{\partial C_L}{\partial \delta_{\mathrm{long}_{\mathrm{LEF}}}}\Delta\delta_{\mathrm{long}_{\mathrm{LEF}}} \\ & + \frac{\partial C_L}{\partial \delta_{\mathrm{long}_{\mathrm{TEF}}}}\Delta\delta_{\mathrm{long}_{\mathrm{TEF}}} + \frac{\partial C_L}{\partial \delta_{\mathrm{lat}_{\mathrm{AIL}}}}\Delta\delta_{\mathrm{lat}_{\mathrm{AIL}}} \\ & + \frac{\partial C_L}{\partial \delta_{\mathrm{lat}_{\mathrm{HT}}}}\Delta\delta_{\mathrm{lat}_{\mathrm{HT}}} + \frac{\partial C_L}{\partial \delta_{\mathrm{lat}_{\mathrm{RUD}}}}\Delta\delta_{\mathrm{lat}_{\mathrm{RUD}}} + \frac{\partial C_L}{\partial \delta_{\mathrm{lat}_{\mathrm{LEF}}}}\Delta\delta_{\mathrm{lat}_{\mathrm{LEF}}} \\ & + \frac{\partial C_L}{\partial \delta_{\mathrm{lat}_{\mathrm{TEF}}}}\Delta\delta_{\mathrm{lat}_{\mathrm{TEF}}} \end{aligned} \quad (5)$$

The variations of all stability coefficients are written under similar forms as the ones given by Eq. (5).

Block 2 description: The variations of forces and moments are written as a function of their stability and control derivatives as follows:

$$\begin{pmatrix} \Delta X_w \\ \Delta Y_w \\ \Delta Z_w \\ \Delta L_w \\ \Delta M_w \\ \Delta N_w \end{pmatrix} = \begin{pmatrix} \bar{S} & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{S} & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{S} & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{S}\cdot b & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{S}\cdot\bar{c} & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{S}\cdot b \end{pmatrix} \begin{pmatrix} \Delta C_D \\ \Delta C_Y \\ \Delta C_{\mathrm{Lift}} \\ \Delta C_L \\ \Delta C_M \\ \Delta C_N \end{pmatrix} \quad (6)$$

where the force variations are $[\Delta X_w \quad \Delta Y_w \quad \Delta Z_w]$, the moment variations are $[\Delta L_w \quad \Delta M_w \quad \Delta N_w]$, and where $\bar{S} = (\rho V^2/2)S = q_{\mathrm{dyn}}S$.

Block 3 description: It is well known in this field [3] that an aircraft's system of coordinates (x_a, y_a, z_a) is related to the wind coordinate system (x_w, y_w, z_w) by the following two successive rotations of the coordinate system (x_w, y_w, z_w): A first rotation with sideslip angle β around the z_w axis to obtain the intermediate coordinate system $(x'y'z')$, and a second rotation with attack angle α around the y' axis to obtain the aircraft coordinate system (x_a, y_a, z_a), which gives

$$\begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix} = \begin{bmatrix} \cos\alpha\cos\beta & -\cos\alpha\sin\beta & -\sin\alpha \\ \sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & -\sin\alpha\sin\beta & \cos\alpha \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} \quad (7)$$

Forces X_a and Z_a have opposite signs, whereas moments L_a, M_a, and N_a have the same sign with respect to the classical formulation [Eq. (8)], as given by NASA DFRC. Then, equations giving X_a, Y_a, Z_a, L_a, M_a, and N_a are linearized around the equilibrium position (*trim*) of the F/A-18 aircraft by the small perturbations theory, in which the index of any quantity at equilibrium is denoted by zero and the variation of a quantity around its equilibrium position is denoted by Δ. Angles of attack and sideslip angles are expressed as $\alpha = \alpha_0 + \Delta\alpha$ and $\beta = \beta_0 + \Delta\beta$. The sideslip angle at equilibrium β_0 given by the NASA DFRC is equal to zero, and therefore we can write the sideslip angle β as a function of its small variation $\Delta\beta$, and we obtain $\cos\beta = \cos(\beta_0 + \Delta\beta) = \cos\Delta\beta = 1$ and $\sin\beta = \sin(\beta_0 + \Delta\beta) = \sin\Delta\beta = \Delta\beta$. The aircraft forces X, Y, Z and moments L, M, N calculated in the aircraft system a and in the wind system w are also written by use of the small perturbations theory, for example, $X_a = X_{a0} + \Delta X_a$, etc. The forces X_0, Y_0, Z_0 and the moments L_0, M_0, N_0 at equilibrium are zeros in the aircraft system a and in the wind system w, whereas the angle of attack $\Delta\alpha$ and sideslip angle $\Delta\beta$ variations are equal to zero. With these last approximations, we obtain the following system of equations:

$$\begin{pmatrix} \Delta X_a \\ \Delta Y_a \\ \Delta Z_a \\ \Delta L_a \\ \Delta M_a \\ \Delta N_a \end{pmatrix} = C_{m1} \begin{pmatrix} \Delta X_w \\ \Delta Y_w \\ \Delta Z_w \\ \Delta L_w \\ \Delta M_w \\ \Delta N_w \end{pmatrix} \quad (8)$$

where C_{m1} has the following form:

$$C_{m1} = \begin{pmatrix} -\cos\alpha_0 & 0 & \sin\alpha_0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\sin\alpha_0 & 0 & -\cos\alpha_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha_0 & 0 & -\sin\alpha_0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \sin\alpha_0 & 0 & \cos\alpha_0 \end{pmatrix} \quad (9)$$

We replace the force and moment variations in the wind system of coordinates ΔX_w, ΔY_w, ΔZ_w, ΔL_w, ΔM_w, and ΔN_w as function of stability coefficients variations given by Eq. (6) into the right-hand side of Eq. (9) and we obtain

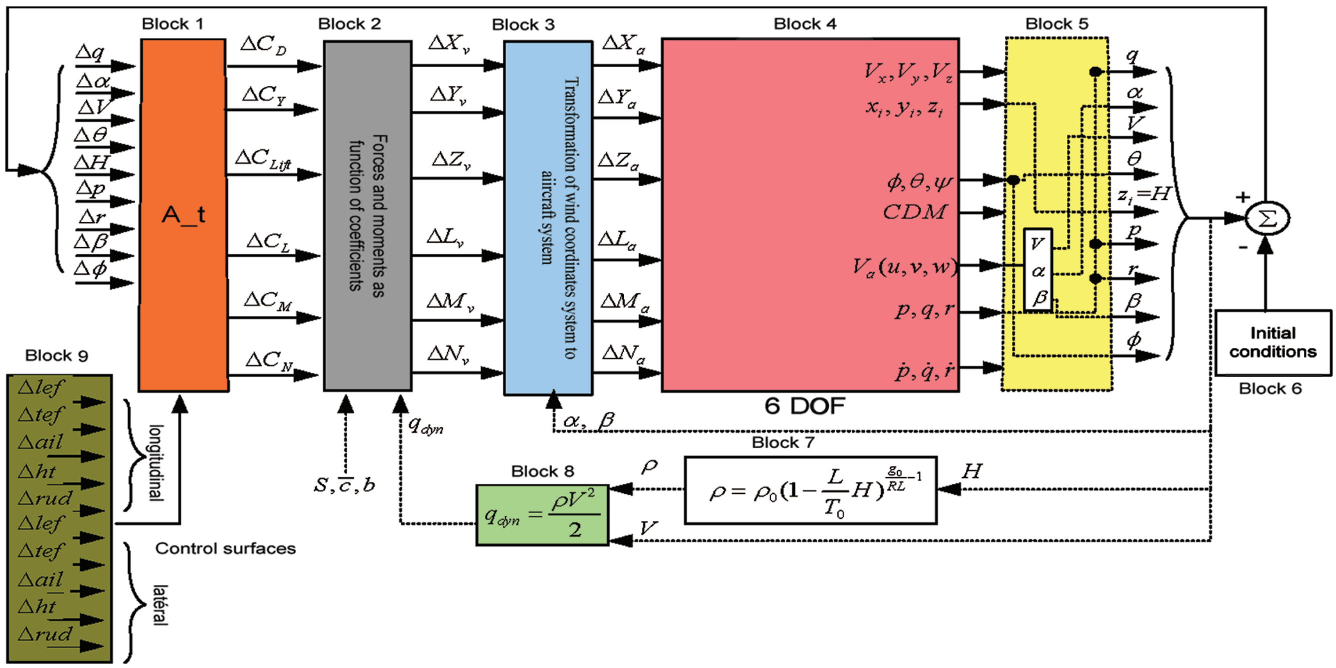

Fig. 1 First simulation scheme with the stability and control coefficients in the wind coordinates system.

$$\begin{pmatrix} \Delta X_a \\ \Delta Y_a \\ \Delta Z_a \\ \Delta L_a \\ \Delta M_a \\ \Delta N_a \end{pmatrix} = C_m \begin{pmatrix} \Delta C_D \\ \Delta C_Y \\ \Delta C_{\mathrm{Lift}} \\ \Delta C_L \\ \Delta C_M \\ \Delta C_N \end{pmatrix} \tag{10}$$

where the C_m matrix has the following form:

$$C_m = \begin{pmatrix} -\bar{S}\cdot\cos\alpha_0 & 0 & \bar{S}\cdot\sin\alpha_0 & 0 & 0 & 0 \\ 0 & \bar{S} & 0 & 0 & 0 & 0 \\ -\bar{S}\cdot\sin\alpha_0 & 0 & -\bar{S}\cdot\cos\alpha_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{S}\cdot b\cdot\cos\alpha_0 & 0 & -\bar{S}\cdot b\cdot\sin\alpha_0 \\ 0 & 0 & 0 & 0 & \bar{S}\cdot\bar{c} & 0 \\ 0 & 0 & 0 & \bar{S}\cdot b\cdot\sin\alpha_0 & 0 & \bar{S}\cdot b\cdot\cos\alpha_0 \end{pmatrix} \tag{11}$$

Block 4 description: The six degree-of-freedom block refers to the equations of motion of a rigid body in six degrees of freedom. The theory can be found in the literature [3]. The Simulink toolbox is used here as block 4, and is composed of five blocks, 4.1–4.5.

Blocks 4.1 and 4.2 descriptions: The origin of the aircraft system of coordinates a is the aircraft center of gravity. The inertial system of coordinates is fixed to the Earth and is denoted by i. The aircraft equations of motion are obtained with Newton's second law (see Fig. 3): The sum of the external forces acting on an aircraft is equal to the momentum rate of change of the momentum of the aircraft over time (block 4.1). The sum of the external moments [3] acting on an aircraft is equal to the angular momentum rate of change of the aircraft over time (block 4.2).

Block 4.3 description: The cosine director matrix (CDM) is calculated from the Euler roll, pitch, and yaw angles ϕ, θ, and ψ in block 4.3 (see Fig. 4), and is used in block 4.4.

Block 4.4 description: The linear speeds V_x, V_y, and V_z in the inertial system of coordinates i are calculated as a function of the linear speeds u, v, and w in the aircraft systems a by three successive rotations: one first rotation with the yaw angle ψ around the z_a axis, a second rotation with the pitch angle θ around the y_a axis, and a third rotation with the roll angle ϕ around the x_a axis, and we obtain

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \mathrm{CDM}^T \begin{pmatrix} u \\ v \\ w \end{pmatrix} \tag{13}$$

Block 4.5 description: Block 4.5 relates the Euler angles ϕ, θ, and ψ, and the angular speeds p, q, and r, with the Euler time derivatives $\dot{\phi}$, $\dot{\theta}$, and $\dot{\psi}$ by use of the following equation:

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} \tag{14}$$

Block 5 description: In block 5, we calculate the variations of the aircraft's true airspeed V, angle of attack α, and sideslip angle β. To calculate these variations, we calculate V, α, and β as functions of linear speeds in translation u, v, and w in the aircraft system of coordinates a [3]:

$$V = \sqrt{u^2 + v^2 + w^2}; \qquad \alpha = \arctan(w/u); \qquad \beta = \arctan[v/\sqrt{(u^2 + w^2)}] \tag{15}$$

The theory of small perturbations is applied to the terms V, α, β, u, v, and w.

Block 5.1. True airspeed variation ΔV: We replace the speeds given by the perturbation theory in the first squared equation of the system of Eq. (20):

$$(V_0 + \Delta V)^2 = (u_0 + \Delta u)^2 + (v_0 + \Delta v)^2 + (w_0 + \Delta w)^2 \tag{16}$$

$$\mathrm{CDM} = \begin{pmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi & \sin\phi\sin\theta\sin\psi + \cos\psi\cos\phi & \cos\theta\sin\phi \\ \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi & \cos\theta\cos\phi \end{pmatrix}^T \tag{12}$$

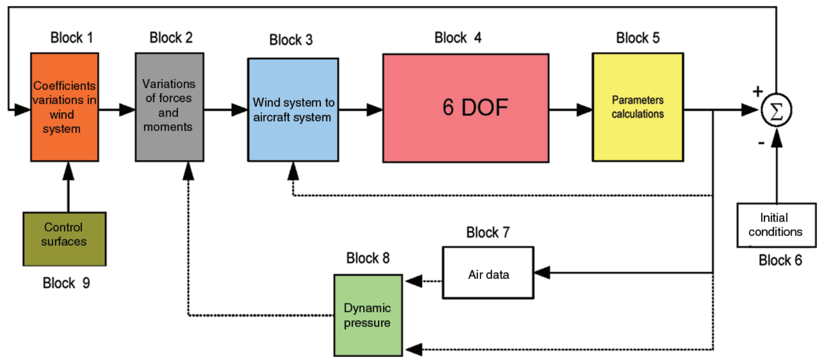

Fig. 2 Simplification of the scheme shown in Fig. 1.

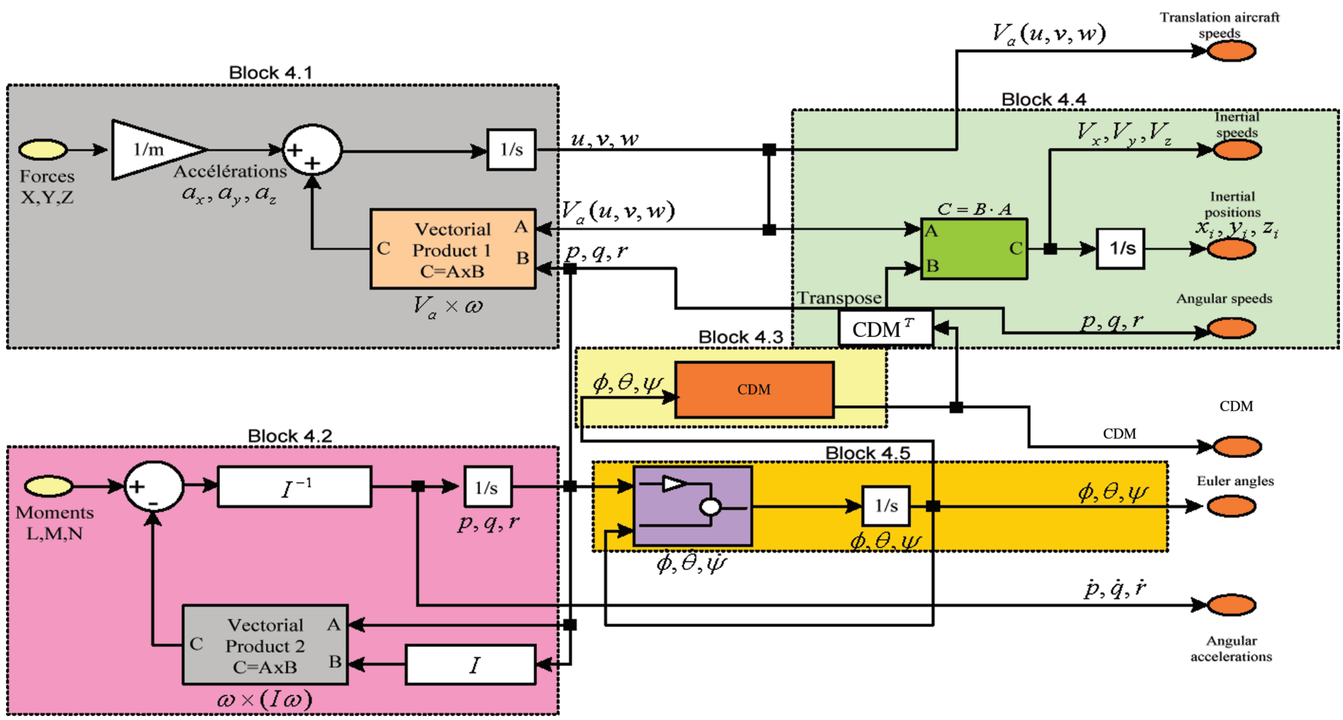

Fig. 3 Description of block 4 details on six degree-of-freedom dynamics.

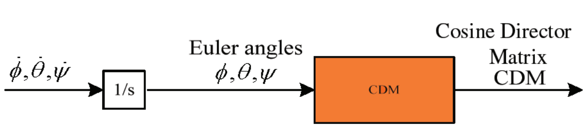

Fig. 4 Scheme of block 4.3.

At equilibrium, we write

$$V_0^2 = u_0^2 + v_0^2 + w_0^2 \tag{17}$$

The square products of speed variations Δu^2, Δv^2, Δw^2, and ΔV^2 in Eq. (16) can be neglected, as they are very small. Then, we divide Eq. (16) by $2V_0$ and we obtain

$$\Delta V = \frac{u_0}{V_0}\Delta u + \frac{v_0}{V_0}\Delta v + \frac{w_0}{V_0}\Delta w \tag{18}$$

At equilibrium, the components u_0 and w_0 of the true airspeed V_0 can be written as a function of the angle of attack α_0 as follows:

$$u_0 = V_0 \cos\alpha_0; \qquad w_0 = V_0 \sin\alpha_0 \tag{19}$$

The researchers at NASA DFRC considered in their calculations $\beta_0 = 0$, so that the component v_0 is equal to zero. We replace Eqs. (19) and $v_0 = 0$ into Eq. (18) and we obtain

$$\Delta V = \Delta u \cos\alpha_0 + \Delta w \sin\alpha_0 \tag{20}$$

Block 5.2. Angle of attack variation ΔV: At equilibrium, the second equation of system (15) can also be written in the following form:

$$u_0 \sin\alpha_0 = w_0 \cos\alpha_0 \tag{21}$$

We apply the perturbation theory to the quantities u, v, and α (for example, $u = u_0 + \Delta u$, etc.). We further replace these quantities, Eqs. (24) and (28) in Eq. (27). The products of small quantity variations, such as $\Delta\alpha\Delta u$ and $\Delta\alpha\Delta w$ are neglected. We take into account the trigonometric equality $\sin^2\alpha_0 + \cos^2\alpha_0 = 1$ and the trigonometric functions for small angles-of-attack variations ($\cos\Delta\alpha = 1$ and $\sin\Delta\alpha = \Delta\alpha$), therefore, we obtain the variation of the angle of attack as follows:

$$\Delta\alpha = \frac{-\sin\alpha_0}{V_0}\Delta u + \frac{\cos\alpha_0}{V_0}\Delta w \tag{22}$$

Block 5.3. Sideslip angle variation $\Delta\beta$: The third equation of the system of Eq. (20) can be written, for small variations of $\Delta\beta_j$ (and for $\beta_0 = 0$), as

$$\tan\Delta\beta \simeq \Delta\beta = \frac{v}{\sqrt{u^2 + w^2}} \tag{23}$$

We express $\Delta\beta$ in the form of the products sum of the β derivatives with respect to u, v, and w at equilibrium and the small perturbations of u, v, and w

$$\Delta\beta = \left(\frac{\partial\beta}{\partial u}\right)_0 \Delta u + \left(\frac{\partial\beta}{\partial v}\right)_0 \Delta v + \left(\frac{\partial\beta}{\partial w}\right)_0 \Delta w \tag{24}$$

The expressions of β derivatives at equilibrium are calculated by the derivation of Eq. (23), where $v_0 = 0$, $V_0^2 = u_0^2 + w_0^2$, and we obtain

$$\Delta\beta = \frac{1}{V_0}\Delta v \tag{25}$$

The variations ΔV, $\Delta\alpha$, and $\Delta\beta$, from Eqs. (20), (22), and (25), respectively, are rearranged in the form of a matrix system of equations:

$$\begin{pmatrix} \Delta V \\ \Delta\alpha \\ \Delta\beta \end{pmatrix} = \begin{pmatrix} \cos\alpha_0 & 0 & \sin\alpha_0 \\ \frac{-\sin\alpha_0}{V_0} & 0 & \frac{\cos\alpha_0}{V_0} \\ 0 & \frac{1}{V_0} & 0 \end{pmatrix} \begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \end{pmatrix} \tag{26}$$

We add the system of Eq. (26) to the initial equations of the block 4 output variations Δu, Δv, Δw, Δx_i, Δy_i, Δz_i, Δp, Δq, Δr, $\Delta\phi$, $\Delta\theta$, and $\Delta\psi$, which become the block 5 inputs, and we obtain a higher-order matrix system of equations:

$$\begin{pmatrix} \Delta q \\ \Delta\alpha \\ \Delta V \\ \Delta\theta \\ \Delta H \\ \Delta p \\ \Delta r \\ \Delta\beta \\ \Delta\phi \end{pmatrix} = B_m \begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \\ \Delta x_i \\ \Delta y_i \\ \Delta z_i = \Delta H \\ \Delta p \\ \Delta q \\ \Delta r \\ \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{\sin\alpha_0}{V_0} & 0 & \frac{\cos\alpha_0}{V_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cos\alpha_0 & 0 & \sin\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{V_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \\ \Delta x_i \\ \Delta y_i \\ \Delta z_i = \Delta H \\ \Delta p \\ \Delta q \\ \Delta r \\ \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{pmatrix} \tag{27}$$

Block 6 description: The initial parameters of the matrix system of Eq. (27) are given by NASA DFRC in the stability and control derivative files for each flight condition characterized by the Mach number, altitude, and angle of attack:

$$\begin{aligned} &[\, q_0 \quad \alpha_0 \quad V_0 \quad \theta_0 \quad H_0 \quad p_0 \quad r_0 \quad \beta_0 \quad \phi_0 \,]^T \\ &\quad = [\, 0 \quad \alpha_{0_{\text{NASA}}} \quad V_{0_{\text{NASA}}} \quad \theta_{0_{\text{NASA}}} \quad H_{0_{\text{NASA}}} \quad 0 \quad 0 \quad 0 \quad 0 \,]^T \end{aligned} \tag{28}$$

Block 7 description: We calculate the air density ρ as a function of altitude H [4].

Block 8 description: The dynamic pressure q_{dyn} is calculated [3] as a function of the aircraft's true airspeed V and the air density ρ.

Block 9 description: The control surface inputs are excited with various signals. The control surfaces for the F/A-18 aircraft analyzed here are as follows: the leading-edge (LE) flaps $\Delta\text{lef} = \Delta\delta_{\text{LEF}}$, the trailing-edge (TE) flaps $\Delta\text{tef} = \Delta\delta_{\text{TEF}}$, the ailerons $\Delta\text{ail} = \Delta\delta_{\text{AIL}}$, the horizontal tail $\Delta\text{ht} = \Delta\delta_{\text{HT}}$, and the rudder $\Delta\text{rud} = \Delta\delta_{\text{RUD}}$. One signal is used at a time on the aircraft control inputs, to give a deviation of $\pm 5^0$ around the equilibrium aircraft position (trim) for its longitudinal and lateral aircraft motion to observe the forces and moments variation over time. For the longitudinal motion, a signal was given on the horizontal tail, and for the lateral motion, a signal was given on the ailerons.

III. State Variables Introduction in the Closed Loop

We next develop a second formulation from the first formulation, to obtain the vectors of the generalized coordinates and their time derivatives in closed loop form:

$$\begin{aligned} \eta &= (\Delta x_i, \Delta y_i, \Delta z_i, \Delta\phi, \Delta\theta, \Delta\psi)^T; \\ \dot{\eta} &= (\Delta V_x, \Delta V_y, \Delta V_z, \Delta\dot{\phi}, \Delta\dot{\theta}, \Delta\dot{\psi})^T \end{aligned} \tag{29}$$

The arrangement of stability coefficients on lines 1–6 of the matrices used in the first formulation is C_L, C_M, C_N, C_D, C_{Lift}, C_Y [see Eq. (6)]. To obtain the second formulation, it is necessary to rearrange the order of the stability coefficients as follows: C_D, C_Y, C_{Lift}, C_L, C_M, C_N. Therefore, the notations A_{int}, $B_{\text{long}_{\text{int}}}$, and $B_{\text{lat}_{\text{int}}}$ are introduced for the intermediate matrices A, B_{long}, and B_{lat}, in which the order of the coefficients is rearranged.

Next, we show only the A_{int} matrix, whereas the $B_{\text{long}_{\text{int}}}$ (6×6) matrix contains the derivatives of the same coefficients as the ones of A_{int} with respect to the longitudinal control surfaces $\delta_{\text{long}_{\text{AIL}}}$, $\delta_{\text{long}_{\text{HT}}}$, $\delta_{\text{long}_{\text{RUD}}}$, $\delta_{\text{long}_{\text{LEF}}}$, and $\delta_{\text{long}_{\text{TEF}}}$, and $B_{\text{lat}_{\text{NASA}}}$ is a (6×6) matrix containing the derivatives of the same coefficients as the ones of A_{int} with respect to the lateral control surfaces $\delta_{\text{lat}_{\text{AIL}}}$, $\delta_{\text{lat}_{\text{HT}}}$, $\delta_{\text{lat}_{\text{RUD}}}$, $\delta_{\text{lat}_{\text{LEF}}}$, and $\delta_{\text{lat}_{\text{TEF}}}$.

$$A_{\text{int}} = \begin{bmatrix} \frac{\partial C_D}{\partial q} & \frac{\partial C_D}{\partial \alpha} & \frac{\partial C_D}{\partial V} & \frac{\partial C_D}{\partial \theta} & \frac{\partial C_D}{\partial H} & \frac{\partial C_D}{\partial p} & \frac{\partial C_D}{\partial r} & \frac{\partial C_D}{\partial \beta} & \frac{\partial C_D}{\partial \varphi} \\ \frac{\partial C_Y}{\partial q} & \frac{\partial C_Y}{\partial \alpha} & \frac{\partial C_Y}{\partial V} & \frac{\partial C_Y}{\partial \theta} & \frac{\partial C_Y}{\partial H} & \frac{\partial C_Y}{\partial p} & \frac{\partial C_Y}{\partial r} & \frac{\partial C_Y}{\partial \beta} & \frac{\partial C_Y}{\partial \varphi} \\ \frac{\partial C_{\text{Lift}}}{\partial q} & \frac{\partial C_{\text{Lift}}}{\partial \alpha} & \frac{\partial C_{\text{Lift}}}{\partial V} & \frac{\partial C_{\text{Lift}}}{\partial \theta} & \frac{\partial C_{\text{Lift}}}{\partial H} & \frac{\partial C_{\text{Lift}}}{\partial p} & \frac{\partial C_{\text{Lift}}}{\partial r} & \frac{\partial C_{\text{Lift}}}{\partial \beta} & \frac{\partial C_{\text{Lift}}}{\partial \varphi} \\ \frac{\partial C_L}{\partial q} & \frac{\partial C_L}{\partial \alpha} & \frac{\partial C_L}{\partial V} & \frac{\partial C_L}{\partial \theta} & \frac{\partial C_L}{\partial H} & \frac{\partial C_L}{\partial p} & \frac{\partial C_L}{\partial r} & \frac{\partial C_L}{\partial \beta} & \frac{\partial C_L}{\partial \varphi} \\ \frac{\partial C_M}{\partial q} & \frac{\partial C_M}{\partial \alpha} & \frac{\partial C_M}{\partial V} & \frac{\partial C_M}{\partial \theta} & \frac{\partial C_M}{\partial H} & \frac{\partial C_M}{\partial p} & \frac{\partial C_M}{\partial r} & \frac{\partial C_M}{\partial \beta} & \frac{\partial C_M}{\partial \varphi} \\ \frac{\partial C_N}{\partial q} & \frac{\partial C_N}{\partial \alpha} & \frac{\partial C_N}{\partial V} & \frac{\partial C_N}{\partial \theta} & \frac{\partial C_N}{\partial H} & \frac{\partial C_N}{\partial p} & \frac{\partial C_N}{\partial r} & \frac{\partial C_N}{\partial \beta} & \frac{\partial C_N}{\partial \varphi} \end{bmatrix} \tag{30}$$

The intermediate matrix $A_{m_{\text{int}}}$ for the A_m matrix is now written as follows:

$$\begin{aligned} A_{m_{\text{int}}} &= [\, A_{\text{int}} \quad B_{\text{long}_{\text{int}}} \quad B_{\text{lat}_{\text{int}}} \,] = \begin{bmatrix} a_{4,1} & a_{4,2} & \dots & a_{4,19} \\ a_{6,1} & a_{6,2} & \dots & a_{6,19} \\ a_{5,1} & a_{5,2} & \dots & a_{5,19} \\ a_{1,1} & a_{1,2} & \dots & a_{1,19} \\ a_{2,1} & a_{2,2} & \dots & a_{2,19} \\ a_{3,1} & a_{3,2} & \dots & a_{3,19} \end{bmatrix} \\ &= [\, A_{m1_{\text{int}}} \quad A_{m2_{\text{int}}} \,] \end{aligned} \tag{31}$$

where the $A_{m1_{\text{int}}}$ matrix of dimensions (6×9) is the stability coefficients matrix, whereas the $A_{m2_{\text{int}}}$ matrix of dimensions (6×10) is the control coefficients matrix $[\, B_{\text{long}_{\text{int}}} \quad B_{\text{lat}_{\text{int}}} \,]$. Then, the second equation of a state-space system may be written in the rearranged order:

$$\begin{pmatrix} \Delta C_D \\ \Delta C_Y \\ \Delta C_{\text{Lift}} \\ \Delta C_L \\ \Delta C_M \\ \Delta C_N \end{pmatrix} = A_{m1_{\text{int}}} \begin{pmatrix} \Delta q \\ \Delta\alpha \\ \Delta V \\ \Delta\theta \\ \Delta H \\ \Delta p \\ \Delta r \\ \Delta\beta \\ \Delta\phi \end{pmatrix} + A_{m2_{\text{int}}} \begin{pmatrix} \Delta\delta_{\text{long}_{\text{LEF}}} \\ \Delta\delta_{\text{long}_{\text{TEF}}} \\ \Delta\delta_{\text{long}_{\text{AIL}}} \\ \Delta\delta_{\text{long}_{\text{HT}}} \\ \Delta\delta_{\text{long}_{\text{RUD}}} \\ \Delta\delta_{\text{lat}_{\text{LEF}}} \\ \Delta\delta_{\text{lat}_{\text{TEF}}} \\ \Delta\delta_{\text{lat}_{\text{AIL}}} \\ \Delta\delta_{\text{lat}_{\text{HT}}} \\ \Delta\delta_{\text{lat}_{\text{RUD}}} \end{pmatrix} \tag{32}$$

We replace the left-hand side term of Eq. (27) in the first term on the right-hand side of Eq. (32) to obtain another set of state vectors $\mathbf{x}$ in the second equation of the state-space system of equations, and we obtain

$$\begin{pmatrix} \Delta C_D \\ \Delta C_Y \\ \Delta C_{\text{Lift}} \\ \Delta C_L \\ \Delta C_M \\ \Delta C_N \end{pmatrix} = A_{m1_{\text{int}}} B_m \begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \\ \Delta x_i \\ \Delta y_i \\ \Delta z_i = \Delta H \\ \Delta p \\ \Delta q \\ \Delta r \\ \Delta \phi \\ \Delta \theta \\ \Delta \psi \end{pmatrix} + A_{m2_{\text{int}}} \begin{pmatrix} \Delta \delta_{\text{long}_{\text{LEF}}} \\ \Delta \delta_{\text{long}_{\text{TEF}}} \\ \Delta \delta_{\text{long}_{\text{AIL}}} \\ \Delta \delta_{\text{long}_{\text{HT}}} \\ \Delta \delta_{\text{long}_{\text{RUD}}} \\ \Delta \delta_{\text{lat}_{\text{LEF}}} \\ \Delta \delta_{\text{lat}_{\text{TEF}}} \\ \Delta \delta_{\text{lat}_{\text{AIL}}} \\ \Delta \delta_{\text{lat}_{\text{HT}}} \\ \Delta \delta_{\text{lat}_{\text{RUD}}} \end{pmatrix} \tag{33}$$

The scheme shown in Fig. 5 represents the conversion of the scheme shown in Fig. 1 by use of Eqs. (29–36), and has the 12 state vectors **x** given by Eq. (29) in the closed loop. We obtained the C and D matrices, which characterize the linear system at trim condition. These matrices are presented in detail next.

We represent the terms of the C and D matrices analytically. The C matrix has the following analytical form:

$$C = C_m A_{m1} B_m = \begin{pmatrix} C11 & C12 & C13 & C14 \\ C21 & C22 & C23 & C24 \end{pmatrix} \tag{37}$$

where the $C11$, $C12$, $C13$, $C14$, $C21$, $C22$, $C23$, and $C24$ matrices are represented under analytical forms. We obtain

$$A_{m1} B_m = \begin{pmatrix} -a_{4,2}\frac{\sin\alpha_0}{V_0} + a_{4,3}\cos\alpha_0 & \frac{a_{4,8}}{V_0} & a_{4,2}\frac{\cos\alpha_0}{V_0} + a_{4,3}\sin\alpha_0 & 0 & 0 & a_{4,5} & a_{4,6} & a_{4,1} & a_{4,7} & a_{4,9} & a_{4,4} & 0 \\ -a_{6,2}\frac{\sin\alpha_0}{V_0} + a_{6,3}\cos\alpha_0 & \frac{a_{6,8}}{V_0} & a_{6,2}\frac{\cos\alpha_0}{V_0} + a_{6,3}\sin\alpha_0 & 0 & 0 & a_{6,5} & a_{6,6} & a_{6,1} & a_{6,7} & a_{6,9} & a_{6,4} & 0 \\ -a_{5,2}\frac{\sin\alpha_0}{V_0} + a_{5,3}\cos\alpha_0 & \frac{a_{5,8}}{V_0} & a_{5,2}\frac{\cos\alpha_0}{V_0} + a_{5,3}\sin\alpha_0 & 0 & 0 & a_{5,5} & a_{5,6} & a_{5,1} & a_{5,7} & a_{5,9} & a_{5,4} & 0 \\ -a_{1,2}\frac{\sin\alpha_0}{V_0} + a_{1,3}\cos\alpha_0 & \frac{a_{1,8}}{V_0} & a_{1,2}\frac{\cos\alpha_0}{V_0} + a_{1,3}\sin\alpha_0 & 0 & 0 & a_{1,5} & a_{1,6} & a_{1,1} & a_{1,7} & a_{1,9} & a_{1,4} & 0 \\ -a_{2,2}\frac{\sin\alpha_0}{V_0} + a_{2,3}\cos\alpha_0 & \frac{a_{2,8}}{V_0} & a_{2,2}\frac{\cos\alpha_0}{V_0} + a_{2,3}\sin\alpha_0 & 0 & 0 & a_{2,5} & a_{2,6} & a_{2,1} & a_{2,7} & a_{2,9} & a_{2,4} & 0 \\ -a_{3,2}\frac{\sin\alpha_0}{V_0} + a_{3,3}\cos\alpha_0 & \frac{a_{3,8}}{V_0} & a_{3,2}\frac{\cos\alpha_0}{V_0} + a_{3,3}\sin\alpha_0 & 0 & 0 & a_{3,5} & a_{3,6} & a_{3,1} & a_{3,7} & a_{3,9} & a_{3,4} & 0 \end{pmatrix} \tag{38}$$

We replace the vector output given on the left-hand side of Eq. (33) in the right-hand side term of Eq. (10) to calculate the forces and moments in the aircraft system of coordinates:

$$\begin{pmatrix} \Delta X_a \\ \Delta Y_a \\ \Delta Z_a \\ \Delta L_a \\ \Delta M_a \\ \Delta N_a \end{pmatrix} = C_m A_{m1_{\text{int}}} B_m \begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \\ \Delta x_i \\ \Delta y_i \\ \Delta z_i = \Delta H \\ \Delta p \\ \Delta q \\ \Delta r \\ \Delta \phi \\ \Delta \theta \\ \Delta \psi \end{pmatrix} + C_m A_{m2_{\text{int}}} \begin{pmatrix} \Delta \delta_{\text{long}_{\text{LEF}}} \\ \Delta \delta_{\text{long}_{\text{TEF}}} \\ \Delta \delta_{\text{long}_{\text{AIL}}} \\ \Delta \delta_{\text{long}_{\text{HT}}} \\ \Delta \delta_{\text{long}_{\text{RUD}}} \\ \Delta \delta_{\text{lat}_{\text{LEF}}} \\ \Delta \delta_{\text{lat}_{\text{TEF}}} \\ \Delta \delta_{\text{lat}_{\text{AIL}}} \\ \Delta \delta_{\text{lat}_{\text{HT}}} \\ \Delta \delta_{\text{lat}_{\text{RUD}}} \end{pmatrix} \tag{34}$$

We introduce the following notations:

$$C = C_m A_{m1_{\text{int}}} B_m; \qquad D = C_m A_{m2_{\text{int}}} \tag{35}$$

$$\begin{pmatrix} \Delta X_a \\ \Delta Y_a \\ \Delta Z_a \\ \Delta L_a \\ \Delta M_a \\ \Delta N_a \end{pmatrix} = C \begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \\ \Delta x_i \\ \Delta y_i \\ \Delta z_i = H \\ \Delta p \\ \Delta q \\ \Delta r \\ \Delta \phi \\ \Delta \theta \\ \Delta \psi \end{pmatrix} + D \begin{pmatrix} \Delta \delta_{\text{long}_{\text{LEF}}} \\ \Delta \delta_{\text{long}_{\text{TEF}}} \\ \Delta \delta_{\text{long}_{\text{AIL}}} \\ \Delta \delta_{\text{long}_{\text{HT}}} \\ \Delta \delta_{\text{long}_{\text{RUD}}} \\ \Delta \delta_{\text{lat}_{\text{LEF}}} \\ \Delta \delta_{\text{lat}_{\text{TEF}}} \\ \Delta \delta_{\text{lat}_{\text{AIL}}} \\ \Delta \delta_{\text{lat}_{\text{HT}}} \\ \Delta \delta_{\text{lat}_{\text{RUD}}} \end{pmatrix} \tag{36}$$

The C matrix is the product of the C_m matrix given by Eq. (11) and the $A_{m1}B_m$ matrices' product given by Eq. (38), whereas the C matrix analytical elements are found by identification. Because of the pages number limitations, we give here only the first element of the C11 matrix:

$$c_{1,1} = \bar{S}\left(\frac{a_{4,2}\sin 2\alpha_0}{2V_0} - a_{4,3}\cos^2\alpha_0 - \frac{a_{5,2}\sin^2\alpha_0}{V_0} + \frac{a_{5,3}\sin 2\alpha_0}{2}\right) \tag{39}$$

The D matrix may be written under the form $D = [D1 \quad D2]^T$, where $D1$ and $D2$ are

$$D1 = \begin{pmatrix} d_{1,1} & d_{1,2} & d_{1,3} & d_{1,4} & d_{1,5} & d_{1,6} & d_{1,7} & d_{1,8} & d_{1,9} & d_{1,10} \\ d_{2,1} & d_{2,2} & d_{2,3} & d_{2,4} & d_{2,5} & d_{2,6} & d_{2,7} & d_{2,8} & d_{2,9} & d_{2,10} \\ d_{3,1} & d_{3,2} & d_{3,3} & d_{3,4} & d_{3,5} & d_{3,6} & d_{3,7} & d_{3,8} & d_{3,9} & d_{3,10} \end{pmatrix} \tag{40a}$$

$$D2 = \begin{pmatrix} d_{4,1} & d_{4,2} & d_{4,3} & d_{4,4} & d_{4,5} & d_{4,6} & d_{4,7} & d_{4,8} & d_{4,9} & d_{4,10} \\ d_{5,1} & d_{5,2} & d_{5,3} & d_{5,4} & d_{5,5} & d_{5,6} & d_{5,7} & d_{5,8} & d_{5,9} & d_{5,10} \\ d_{6,1} & d_{6,2} & d_{6,3} & d_{6,4} & d_{6,5} & d_{6,6} & d_{6,7} & d_{6,8} & d_{6,9} & d_{6,10} \end{pmatrix} \tag{40b}$$

The matrix $D = [D1 \quad D2]^T = C_m A_{m2_{\text{int}}}$ is the product of the C_m matrix given by Eq. (11) and the $A_{m2_{\text{int}}}$ matrix. The elements of matrices $D1$ and $D2$ are calculated by identification, and we show here only the first elements of the $D1$ matrix.

$$
\begin{aligned}
d_{1,1} &= \bar{S}\cdot(-a_{4,10}\cdot\cos\alpha_0 + a_{5,10}\cdot\sin\alpha_0);\\
d_{1,2} &= \bar{S}\cdot(-a_{4,11}\cdot\cos\alpha_0 + a_{5,11}\cdot\sin\alpha_0)\\
d_{1,3} &= \bar{S}\cdot(-a_{4,12}\cdot\cos\alpha_0 + a_{5,12}\cdot\sin\alpha_0);\\
d_{1,4} &= \bar{S}\cdot(-a_{4,13}\cdot\cos\alpha_0 + a_{5,13}\cdot\sin\alpha_0)\\
d_{1,5} &= \bar{S}\cdot(-a_{4,14}\cdot\cos\alpha_0 + a_{5,14}\cdot\sin\alpha_0);\\
d_{1,6} &= \bar{S}\cdot(-a_{4,15}\cdot\cos\alpha_0 + a_{5,15}\cdot\sin\alpha_0)\\
d_{1,7} &= \bar{S}\cdot(-a_{4,16}\cdot\cos\alpha_0 + a_{5,16}\cdot\sin\alpha_0);\\
d_{1,8} &= \bar{S}\cdot(-a_{4,17}\cdot\cos\alpha_0 + a_{5,17}\cdot\sin\alpha_0)\\
d_{1,9} &= \bar{S}\cdot(-a_{4,18}\cdot\cos\alpha_0 + a_{5,18}\cdot\sin\alpha_0);\\
d_{1,10} &= \bar{S}\cdot(-a_{4,19}\cdot\cos\alpha_0 + a_{5,19}\cdot\sin\alpha_0)
\end{aligned}
\tag{41}
$$

The C matrix [only the first element of C11 matrix is shown in Eq. (39)] and the D matrix [only its first elements are shown in Eqs. (42)] are further replaced in Eq. (36). Therefore, we obtain the two matrix equations for the variations of forces ΔX_a, ΔY_a, and ΔZ_a and moments ΔL_a, ΔM_a, and ΔN_a in the aircraft system of coordinates:

$$
\begin{pmatrix}\Delta X_a\\ \Delta Y_a\\ \Delta Z_a\end{pmatrix} = C11\begin{pmatrix}\Delta u\\ \Delta v\\ \Delta w\end{pmatrix} + C12\begin{pmatrix}\Delta x_i\\ \Delta y_i\\ \Delta z_i\end{pmatrix} + C13\begin{pmatrix}\Delta p\\ \Delta q\\ \Delta r\end{pmatrix} + C14\begin{pmatrix}\Delta\phi\\ \Delta\theta\\ \Delta\psi\end{pmatrix} + D1\begin{pmatrix}\Delta\delta_{\mathrm{long_{LEF}}}\\ \Delta\delta_{\mathrm{long_{TEF}}}\\ \Delta\delta_{\mathrm{long_{AIL}}}\\ \Delta\delta_{\mathrm{long_{HT}}}\\ \Delta\delta_{\mathrm{long_{RUD}}}\\ \Delta\delta_{\mathrm{lat_{LEF}}}\\ \Delta\delta_{\mathrm{lat_{TEF}}}\\ \Delta\delta_{\mathrm{lat_{AIL}}}\\ \Delta\delta_{\mathrm{lat_{HT}}}\\ \Delta\delta_{\mathrm{lat_{RUD}}}\end{pmatrix}
\tag{42a}
$$

$$
\begin{pmatrix}\Delta L_a\\ \Delta M_a\\ \Delta N_a\end{pmatrix} = C21\begin{pmatrix}\Delta u\\ \Delta v\\ \Delta w\end{pmatrix} + C22\begin{pmatrix}\Delta x_i\\ \Delta y_i\\ \Delta z_i\end{pmatrix} + C23\begin{pmatrix}\Delta p\\ \Delta q\\ \Delta r\end{pmatrix} + C24\begin{pmatrix}\Delta\phi\\ \Delta\theta\\ \Delta\psi\end{pmatrix} + D2\begin{pmatrix}\Delta\delta_{\mathrm{long_{LEF}}}\\ \Delta\delta_{\mathrm{long_{TEF}}}\\ \Delta\delta_{\mathrm{long_{AIL}}}\\ \Delta\delta_{\mathrm{long_{HT}}}\\ \Delta\delta_{\mathrm{long_{RUD}}}\\ \Delta\delta_{\mathrm{lat_{LEF}}}\\ \Delta\delta_{\mathrm{lat_{TEF}}}\\ \Delta\delta_{\mathrm{lat_{AIL}}}\\ \Delta\delta_{\mathrm{lat_{HT}}}\\ \Delta\delta_{\mathrm{lat_{RUD}}}\end{pmatrix}
\tag{42b}
$$

In Eqs. (42a) and (42b), there are terms u, v, w, p, q, and r in the aircraft coordinate system and terms x_i, y_i, z_i, ϕ, θ, and ψ in the inertial coordinate system. We need to obtain these terms in the same system of coordinates, and we choose the inertial system of coordinates. Therefore, we convert the linear and angular speeds terms ($u, v, w, p, q,$ and r) from the aircraft coordinates system a, into the inertial coordinate system i. The speeds u, v, and w in the aircraft system of coordinates a are obtained, using linearization, from the linear speeds and Euler angles in the inertial system of coordinates i. Equation (13) can be written in the following form:

$$
\begin{pmatrix}u\\ v\\ w\end{pmatrix}_a = \begin{pmatrix}\cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta\\ \sin\phi\sin\theta\cos\psi-\cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi+\cos\phi\cos\psi & \sin\phi\cos\theta\\ \cos\phi\sin\theta\cos\psi+\sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi-\sin\phi\cos\psi & \cos\phi\cos\theta\end{pmatrix}\begin{pmatrix}V_x\\ V_y\\ V_z\end{pmatrix}_i
\tag{43}
$$

The small perturbations theory is applied to the following quantities: $u, v, w, V_x, V_y, V_z, p, q,$ and r. The roll and yaw angles at equilibrium given by NASA DFRC are equal to zero, ϕ_0, and ψ_0. The trigonometric function (sine and cosine) assumptions for small angles $\Delta\phi$, $\Delta\theta$, and $\Delta\psi$ are also considered. The products of small terms such as $\Delta\phi\Delta\psi$, $\Delta\theta\Delta V_x$, $\Delta\theta\Delta V_z$, $\Delta\phi\Delta V_x$, and $\Delta\phi\Delta V_y$ are neglected, and we consider the equilibrium speeds V_{y_0} and V_{z_0} to be zero. We replace these assumptions in Eq. (43) and we obtain

$$
\begin{pmatrix}\Delta u\\ \Delta v\\ \Delta w\end{pmatrix} = P\begin{pmatrix}\Delta V_x\\ \Delta V_y\\ \Delta V_z\end{pmatrix} + P1\begin{pmatrix}\Delta\phi\\ \Delta\theta\\ \Delta\psi\end{pmatrix}
\tag{44}
$$

where

$$
P = \begin{pmatrix}\cos\theta_0 & 0 & -\sin\theta_0\\ 0 & 1 & 0\\ \sin\theta_0 & 0 & \cos\theta_0\end{pmatrix} \quad \text{and} \quad P1 = \begin{pmatrix}0 & -V_{x_0}\sin\theta_0 & 0\\ V_{x_0}\sin\theta_0 & 0 & -V_{x_0}\\ 0 & V_{x_0}\cos\theta_0 & 0\end{pmatrix}
\tag{45}
$$

The second type of linearization concerns the calculations of angular speeds p, q, and r in the aircraft system of coordinates from the Euler angles and their derivatives in the inertial system of coordinates. The angular speeds p, q, and r are expressed [3,4] as functions of Euler angles ϕ, θ, and ψ, respectively, and their time derivatives $\dot\phi$, $\dot\theta$, and $\dot\psi$. The small perturbations theory is applied to the Euler angles ϕ, θ, and ψ, their time derivatives $\dot\phi$, $\dot\theta$, and $\dot\psi$, and to the angular speeds p, q, and r. The initial data given by the NASA DFRC laboratories are given for the angular speeds $p_0 = q_0 = r_0 = 0$, Euler angles and their time derivatives $\phi_0 = \psi_0 = 0$, $\theta_0 = \alpha_0$, and $\dot\phi_0 = \dot\theta_0 = \dot\psi_0 = 0$. The products of small angle variations, such as $\Delta\phi\Delta\theta$ and other products of such small angles, can be neglected. For small angle variations $\Delta\phi$, $\Delta\theta$, and $\Delta\psi$ ($\sin\Delta\phi = \Delta\phi, \cos\Delta\phi = 1, \ldots$), we can write the following system of equations:

$$
\begin{pmatrix}\Delta p\\ \Delta q\\ \Delta r\end{pmatrix} = \begin{pmatrix}\Delta\dot\phi - \Delta\dot\psi\sin\theta_0\\ \Delta\dot\theta\\ \cos\theta_0\Delta\dot\psi\end{pmatrix} = \begin{pmatrix}1 & 0 & -\sin\theta_0\\ 0 & 1 & 0\\ 0 & 0 & \cos\theta_0\end{pmatrix}\begin{pmatrix}\Delta\dot\phi\\ \Delta\dot\theta\\ \Delta\dot\psi\end{pmatrix} = R\begin{pmatrix}\Delta\dot\phi\\ \Delta\dot\theta\\ \Delta\dot\psi\end{pmatrix}
\tag{46}
$$

where R is the following matrix:

$$
R = \begin{pmatrix}1 & 0 & -\sin\theta_0\\ 0 & 1 & 0\\ 0 & 0 & \cos\theta_0\end{pmatrix}
\tag{47}
$$

We next replace the vectors $(\Delta u\Delta v\Delta w)^T$ given by Eqs. (44) and (45) and $(\Delta p\Delta q\Delta r)^T$ given by Eqs. (46) and (47) into the two sets of Eqs. (42a) and (42b) for the variations of forces ΔX_a, ΔY_a, ΔZ_a and moments ΔL_a, ΔM_a, ΔN_a in the aircraft system of coordinates, and we obtain

$$\begin{pmatrix} \Delta X_a \\ \Delta Y_a \\ \Delta Z_a \end{pmatrix} = C11 \cdot P \cdot \begin{pmatrix} \Delta V_x \\ \Delta V_y \\ \Delta V_z \end{pmatrix} + C12 \cdot \begin{pmatrix} \Delta X_i \\ \Delta Y_i \\ \Delta Z_i \end{pmatrix} + C13 \cdot R \cdot \begin{pmatrix} \Delta \dot{\phi} \\ \Delta \dot{\theta} \\ \Delta \dot{\psi} \end{pmatrix} + (C14 + C11 \cdot P1) \cdot \begin{pmatrix} \Delta \phi \\ \Delta \theta \\ \Delta \psi \end{pmatrix} + D1 \cdot \begin{pmatrix} \Delta\delta_{\mathrm{long_{LEF}}} \\ \Delta\delta_{\mathrm{long_{TEF}}} \\ \Delta\delta_{\mathrm{long_{AIL}}} \\ \Delta\delta_{\mathrm{long_{HT}}} \\ \Delta\delta_{\mathrm{long_{RUD}}} \\ \Delta\delta_{\mathrm{lat_{LEF}}} \\ \Delta\delta_{\mathrm{lat_{TEF}}} \\ \Delta\delta_{\mathrm{lat_{AIL}}} \\ \Delta\delta_{\mathrm{lat_{HT}}} \\ \Delta\delta_{\mathrm{lat_{RUD}}} \end{pmatrix} \tag{48a}$$

$$\begin{pmatrix} \Delta L_a \\ \Delta M_a \\ \Delta N_a \end{pmatrix} = C21 \cdot P \begin{pmatrix} \Delta V_x \\ \Delta V_y \\ \Delta V_z \end{pmatrix} + C22 \begin{pmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{pmatrix} + C23 \cdot R \begin{pmatrix} \Delta \dot{\phi} \\ \Delta \dot{\theta} \\ \Delta \dot{\psi} \end{pmatrix} + (C24 + C21 \cdot P1) \begin{pmatrix} \Delta \phi \\ \Delta \theta \\ \Delta \psi \end{pmatrix} + D2 \begin{pmatrix} \Delta\delta_{\mathrm{long_{LEF}}} \\ \Delta\delta_{\mathrm{long_{TEF}}} \\ \Delta\delta_{\mathrm{long_{AIL}}} \\ \Delta\delta_{\mathrm{long_{HT}}} \\ \Delta\delta_{\mathrm{long_{RUD}}} \\ \Delta\delta_{\mathrm{lat_{LEF}}} \\ \Delta\delta_{\mathrm{lat_{TEF}}} \\ \Delta\delta_{\mathrm{lat_{AIL}}} \\ \Delta\delta_{\mathrm{lat_{HT}}} \\ \Delta\delta_{\mathrm{lat_{RUD}}} \end{pmatrix} \tag{48b}$$

The development of Eqs. (48a) and (48b) can be seen in Fig. 6.

In Fig. 6, we introduce the following notations for the state vector variations $\Delta\eta$ and $\Delta\dot{\eta}$:

$$\Delta\eta = \begin{bmatrix} \Delta x_i & \Delta y_i & \Delta z_i & \Delta\phi & \Delta\theta & \Delta\psi \end{bmatrix}^T; \quad \Delta\dot{\eta} = \begin{bmatrix} \Delta V_x & \Delta V_y & \Delta V_z & \Delta\dot{\phi} & \Delta\dot{\theta} & \Delta\dot{\psi} \end{bmatrix}^T \tag{49}$$

IV. Aerodynamic Forces for Rigid-to-Rigid and Rigid-to-Control Interactions Mode Calculations

To continue this work, we need to calculate the aerodynamic forces and moments variations for rigid-to-rigid Q_{rr} and rigid-to-control Q_{rc} interaction modes in the inertial system i [5]. Linearization of forces and Euler angles from the aircraft system of coordinates a into the inertial system i is realized by use of the small perturbations theory. The inertial forces variations' ΔX_i, ΔY_i, and ΔZ_i are written as functions of force variations in the aircraft system of coordinates X_a, Y_a, and Z_a by use of the CDM in a form similar to the one given by Eq. (13). The small perturbation theories are applied to the Euler angles, then we obtain two sets of equations for the forces and for the moments, under the following form:

$$\begin{pmatrix} \Delta X_i \\ \Delta Y_i \\ \Delta Z_i \end{pmatrix} = F \begin{pmatrix} \Delta X_a \\ \Delta Y_a \\ \Delta Z_a \end{pmatrix} + F1 \begin{pmatrix} \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{pmatrix} = F \begin{pmatrix} \Delta X_a \\ \Delta Y_a \\ \Delta Z_a \end{pmatrix} + 0 = F \begin{pmatrix} \Delta X_a \\ \Delta Y_a \\ \Delta Z_a \end{pmatrix} \tag{50a}$$

and for the moments

$$\begin{pmatrix} \Delta L_i \\ \Delta M_i \\ \Delta N_i \end{pmatrix} = F \begin{pmatrix} \Delta L_a \\ \Delta M_a \\ \Delta N_a \end{pmatrix} + F1 \begin{pmatrix} \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{pmatrix} = F \begin{pmatrix} \Delta L_a \\ \Delta M_a \\ \Delta N_a \end{pmatrix} + 0 = F \begin{pmatrix} \Delta L_a \\ \Delta M_a \\ \Delta N_a \end{pmatrix} \tag{50b}$$

where

$$F = \begin{pmatrix} \cos\theta_0 & 0 & \sin\theta_0 \\ 0 & 1 & 0 \\ -\sin\theta_0 & 0 & \cos\theta_0 \end{pmatrix} \tag{51}$$

The forces and moments variations in the aircraft system of coordinates given by Eqs. (48a) and (48b) are replaced in Eqs. (50a) and (50b), in which a vector of zeros of dimensions (3×10) is added, and therefore, the forces and moments are obtained in the inertial system of coordinates i:

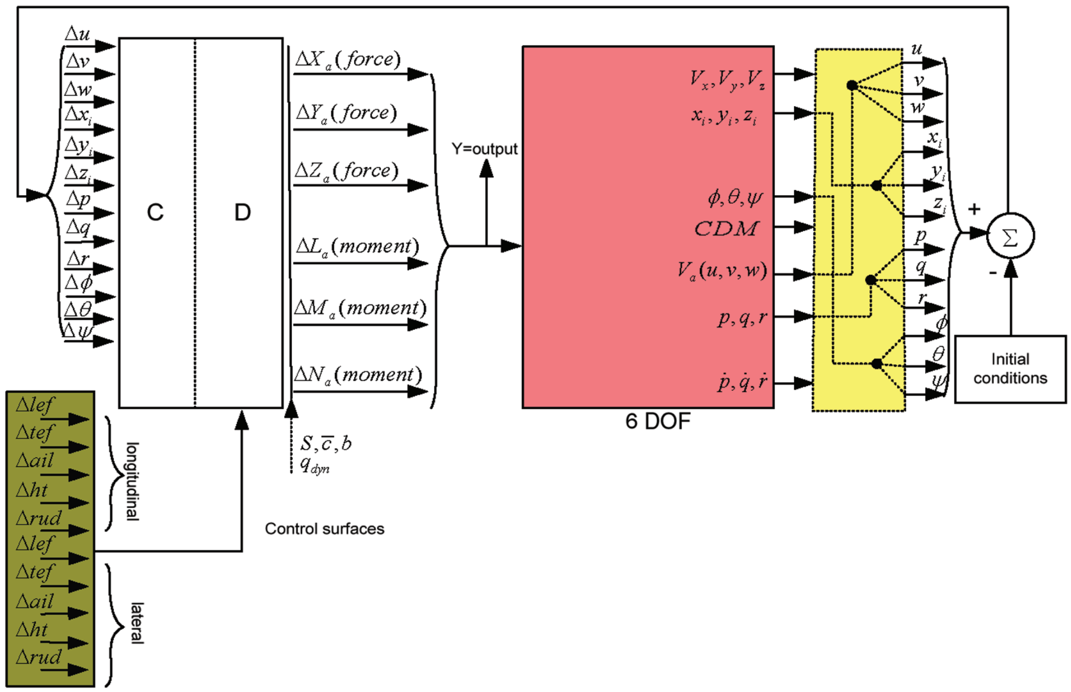

Fig. 5 Simulation scheme with 12 state vectors in closed loop.

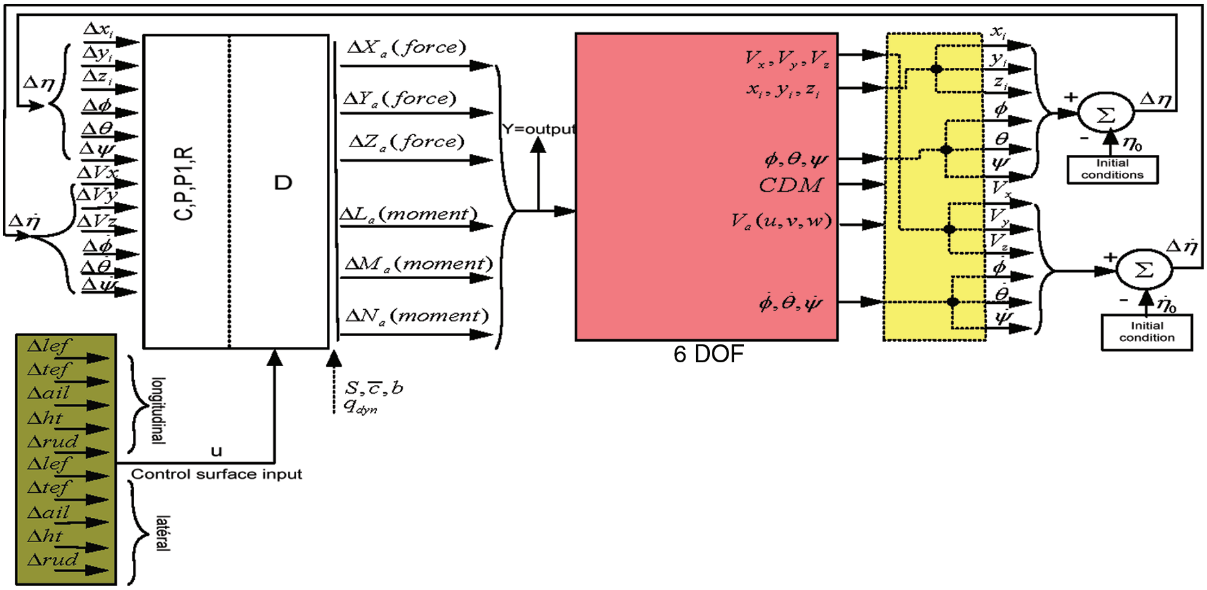

Fig. 6 Scheme including the variations of state vectors and their time derivatives $\Delta\eta$ and $\Delta\dot{\eta}$.

$$\begin{pmatrix}\Delta X_i\\ \Delta Y_i\\ \Delta Z_i\end{pmatrix} = F\cdot C12\cdot\begin{pmatrix}\Delta x_i\\ \Delta y_i\\ \Delta z_i\end{pmatrix} + F(C14+C11\cdot P1)\cdot\begin{pmatrix}\Delta\phi\\ \Delta\theta\\ \Delta\psi\end{pmatrix} + F\cdot D1\cdot\begin{pmatrix}\Delta\delta_{\mathrm{long_{LEF}}}\\ \Delta\delta_{\mathrm{long_{TEF}}}\\ \Delta\delta_{\mathrm{long_{AIL}}}\\ \Delta\delta_{\mathrm{long_{HT}}}\\ \Delta\delta_{\mathrm{long_{RUD}}}\\ \Delta\delta_{\mathrm{lat_{LEF}}}\\ \Delta\delta_{\mathrm{lat_{TEF}}}\\ \Delta\delta_{\mathrm{lat_{AIL}}}\\ \Delta\delta_{\mathrm{lat_{HT}}}\\ \Delta\delta_{\mathrm{lat_{RUD}}}\end{pmatrix} + F\cdot C11\cdot P\cdot\begin{pmatrix}\Delta V_x\\ \Delta V_y\\ \Delta V_z\end{pmatrix} + F\cdot C13\cdot R\cdot\begin{pmatrix}\Delta\dot{\phi}\\ \Delta\dot{\theta}\\ \Delta\dot{\psi}\end{pmatrix} + \mathrm{Zeros}_{(3\times 10)}\cdot(0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)^T \tag{52a}$$

$$\begin{pmatrix}\Delta L_i\\ \Delta M_i\\ \Delta N_i\end{pmatrix} = F\cdot C22\begin{pmatrix}\Delta x_i\\ \Delta y_i\\ \Delta z_i\end{pmatrix} + F(C24+C21\cdot P1)\begin{pmatrix}\Delta\phi\\ \Delta\theta\\ \Delta\psi\end{pmatrix} + F\cdot D2\begin{pmatrix}\Delta\delta_{\mathrm{long_{LEF}}}\\ \Delta\delta_{\mathrm{long_{TEF}}}\\ \Delta\delta_{\mathrm{long_{AIL}}}\\ \Delta\delta_{\mathrm{long_{HT}}}\\ \Delta\delta_{\mathrm{long_{RUD}}}\\ \Delta\delta_{\mathrm{lat_{LEF}}}\\ \Delta\delta_{\mathrm{lat_{TEF}}}\\ \Delta\delta_{\mathrm{lat_{AIL}}}\\ \Delta\delta_{\mathrm{lat_{HT}}}\\ \Delta\delta_{\mathrm{lat_{RUD}}}\end{pmatrix} + F\cdot C21\cdot P\cdot\begin{pmatrix}\Delta V_x\\ \Delta V_y\\ \Delta V_z\end{pmatrix} + F\cdot C23\cdot R\cdot\begin{pmatrix}\Delta\dot{\phi}\\ \Delta\dot{\theta}\\ \Delta\dot{\psi}\end{pmatrix} + \mathrm{Zeros}_{(3\times 10)}\cdot(0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)^T \tag{52b}$$

We arrange Eqs. (52a) and (52b) to obtain the generalized coordinate vectors' variations $\Delta\eta$ and $\Delta\dot{\eta}$ given by Eqs. (49), and the control vectors u and $\dot{u}$.

$$\begin{pmatrix}\Delta X_i\\ \Delta Y_i\\ \Delta Z_i\\ \Delta L_i\\ \Delta M_i\\ \Delta N_i\end{pmatrix} = \begin{bmatrix} FC12 & F(C14+C11P1) & FD1\\ FC22 & F(C24+C21P1) & FD2\end{bmatrix}\begin{pmatrix}\Delta x_i\\ \Delta y_i\\ \Delta z_i\\ \Delta\phi\\ \Delta\theta\\ \Delta\psi\\ \Delta\delta_{\mathrm{long_{LEF}}}\\ \Delta\delta_{\mathrm{long_{TEF}}}\\ \Delta\delta_{\mathrm{long_{AIL}}}\\ \Delta\delta_{\mathrm{long_{HT}}}\\ \Delta\delta_{\mathrm{long_{RUD}}}\\ \Delta\delta_{\mathrm{lat_{LEF}}}\\ \Delta\delta_{\mathrm{lat_{TEF}}}\\ \Delta\delta_{\mathrm{lat_{AIL}}}\\ \Delta\delta_{\mathrm{lat_{HT}}}\\ \Delta\delta_{\mathrm{lat_{RUD}}}\end{pmatrix} + \begin{bmatrix} FC11P & FC13R & 0\\ FC21P & FC23R & 0\end{bmatrix}\begin{pmatrix}\Delta V_x\\ \Delta V_y\\ \Delta V_z\\ \Delta\dot{\phi}\\ \Delta\dot{\theta}\\ \Delta\dot{\psi}\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\end{pmatrix} \tag{53}$$

The system of Eqs. (53) is simulated with the scheme shown in Fig. 7.

To calculate the aerodynamic forces Q_{rr} and Q_{rc}, we need to write the equation of motion for the flexible aircraft structure in terms of generalized coordinates, in the following form:

$$M\ddot{\eta} + D\dot{\eta} + K\eta + q_{\mathrm{dyn}}Q\eta = 0 \tag{54}$$

which can be written in a simplified form as

$$M\ddot{\eta} + D\dot{\eta} + K\eta + y_1 = 0 \tag{55}$$

where the last term of Eq. (55) is:

$$y_1 = F_{\text{aero}} = q_{\text{dyn}}Q\eta = \begin{bmatrix} \Delta X_i & \Delta Y_i & \Delta Z_i & \Delta L_i & \Delta M_i & \Delta N_i \end{bmatrix}^T \tag{56}$$

The aerodynamic forces Q have real and imaginary parts, and for this reason Eq. (56) can also be expressed as

$$y_1 = q_{\text{dyn}}(Q_R + \mathbf{j}Q_I)\eta = q_{\text{dyn}}Q_R\eta + q_{\text{dyn}}\frac{\mathbf{j}\omega\eta}{\omega}Q_I \tag{57}$$

From the generalized coordinates definition $\eta = Ae^{j\omega t}$, where A is the amplitude of motion and ω is the oscillations frequency, we calculate the generalized coordinates derivative with time $\dot{\eta} = \mathbf{j}\omega Ae^{j\omega t} = \mathbf{j}\omega\eta$. The reduced frequency k is given by the following equation:

$$k = \frac{\omega\bar{c}}{2V} \tag{58}$$

We replace ω calculated with Eq. (58) into Eq. (57) and we obtain

$$y_1 = q_{\text{dyn}}Q_R\eta + q_{\text{dyn}}\frac{\dot{\eta}}{\omega}Q_I = q_{\text{dyn}}Q_R\eta + q_{\text{dyn}}\frac{\bar{c}}{2kV}Q_I\dot{\eta} \tag{59}$$

The real parts of aerodynamic forces Q_R correspond to the state vectors $\mathbf{x}$ and control vectors $\mathbf{u}$, and the imaginary parts of aerodynamic forces Q_I correspond to the time derivatives of state vectors $\mathbf{x}$ and control vectors $\mathbf{u}$, and therefore we can write

$$y_1 = q_{\text{dyn}}Q^R_{\text{rr}}\eta + q_{\text{dyn}}Q^R_{\text{rc}}u + q_{\text{dyn}}\frac{\bar{c}}{2Vk}Q^I_{\text{rr}}\dot{\eta} + q_{\text{dyn}}\frac{\bar{c}}{2Vk}Q^I_{\text{rc}}\dot{u} \tag{60}$$

Equation (60) may be expressed in the form of the second state-space equation:

$$y_1 = q_{\text{dyn}}\begin{pmatrix} Q^R_{\text{rr}} & \frac{\bar{c}}{2Vk}Q^I_{\text{rr}} \end{pmatrix}\begin{pmatrix} \eta \\ \dot{\eta} \end{pmatrix} + q_{\text{dyn}}\begin{pmatrix} Q^R_{\text{rc}} & \frac{\bar{c}}{2Vk}Q^I_{\text{rc}} \end{pmatrix}\begin{pmatrix} u \\ \dot{u} \end{pmatrix} \tag{61}$$

Equation (53) may be written by use of state vectors η and their time derivatives $\dot{\eta}$ and with control vectors $\mathbf{u}$ and their time derivatives $\dot{u}$, as follows:

$$y_1 = \begin{pmatrix} \Delta X_i \\ \Delta Y_i \\ \Delta Z_i \\ \Delta L_i \\ \Delta M_i \\ \Delta N_i \end{pmatrix} = \begin{bmatrix} F\cdot C12 & F\cdot(C14 + C11\cdot P1) & F\cdot D1 \\ F\cdot C22 & F\cdot(C24 + C21\cdot P1) & F\cdot D2 \end{bmatrix}\begin{pmatrix} \eta \\ u \end{pmatrix} + \begin{pmatrix} F\cdot C11\cdot P & F\cdot C13\cdot R & 0 \\ F\cdot C21\cdot P & F\cdot C23\cdot R & 0 \end{pmatrix}\begin{pmatrix} \dot{\eta} \\ \dot{u} \end{pmatrix} \tag{62}$$

Equation (61) can also be presented in the following form:

$$y_1 = \begin{bmatrix} \Delta X_i & \Delta Y_i & \Delta Z_i & \Delta L_i & \Delta M_i & \Delta N_i \end{bmatrix}^T = (q_{\text{dyn}}Q^R_{\text{rr}}q_{\text{dyn}}Q^R_{\text{rc}})\begin{pmatrix} \eta \\ u \end{pmatrix} + \left(q_{\text{dyn}}\frac{\bar{c}}{2Vk}Q^I_{\text{rr}}q_{\text{dyn}}\frac{\bar{c}}{2Vk}Q^I_{\text{rc}}\right)\begin{pmatrix} \dot{\eta} \\ \dot{u} \end{pmatrix} \tag{63}$$

The Q_{rr} and Q_{rc} matrices are represented under analytical form (with 6 rows and 16 columns). By identification of the aerodynamic forces matrices given in Eqs. (62) and (63), we calculate the terms of the real aerodynamic forces for rigid-to-rigid mode interactions Q^R_{rr} and the terms of the real aerodynamic forces for rigid-to-control mode interactions Q^R_{rc}, and thus we obtain

$$Q^R_{\text{rr}} = \begin{bmatrix} F\cdot C12 & F\cdot(C14 + C11\cdot P1) \\ F\cdot C22 & F\cdot(C24 + C21\cdot P1) \end{bmatrix} = \begin{pmatrix} Q^R_{\text{rr}_1 1} & Q^R_{\text{rr}_1 2} \\ Q^R_{\text{rr}_2 1} & Q^R_{\text{rr}_2 2} \end{pmatrix} \tag{64a}$$

$$Q^R_{\text{rc}} = \begin{pmatrix} F\cdot D1 \\ F\cdot D2 \end{pmatrix} = \begin{pmatrix} Q^R_{\text{rc}_1 1} \\ Q^R_{\text{rc}_2 1} \end{pmatrix} \tag{64b}$$

The elements of matrices $Q^R_{\text{rr}_1 1}$, $Q^R_{\text{rr}_1 2}$, $Q^R_{\text{rr}_2 1}$, $Q^R_{\text{rr}_2 2}$, $Q^R_{\text{rc}_1 1}$, $Q^R_{\text{rc}_2 1}$ are defined as follows:

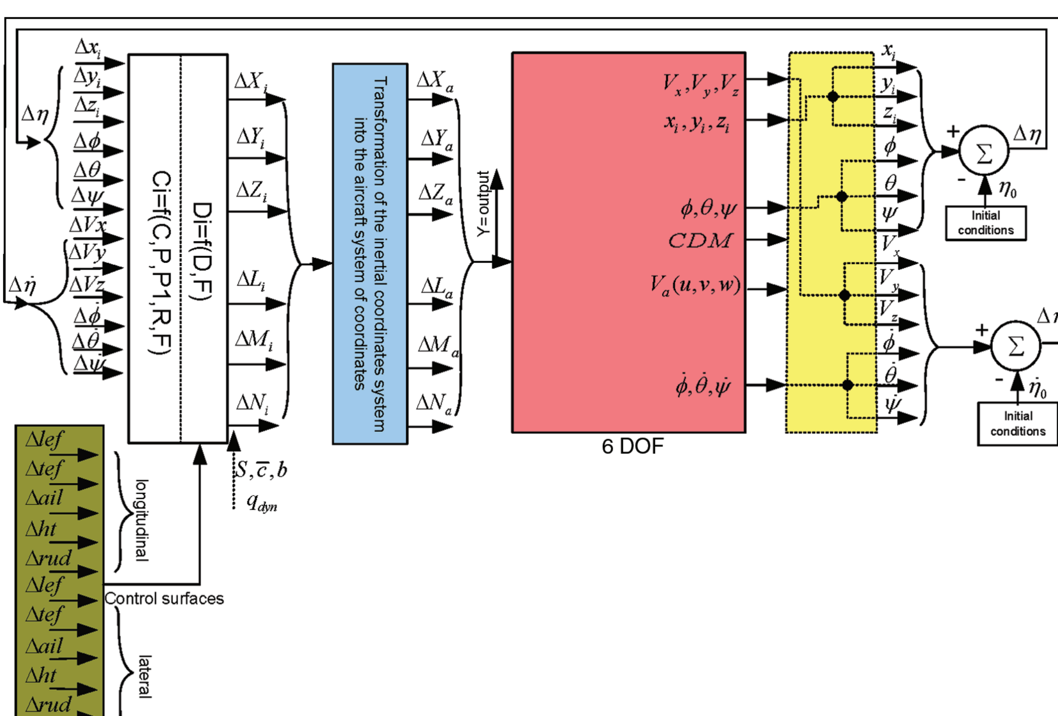

Fig. 7 Scheme with force and moment variations in the aircraft system of coordinates from the inertial system of coordinates.

$$Q^R_{\mathrm{rr}_1 1} = \begin{pmatrix} qr_{1,1} & qr_{1,2} & qr_{1,3} \\ qr_{2,1} & qr_{2,2} & qr_{2,3} \\ qr_{3,1} & qr_{3,2} & qr_{3,3} \end{pmatrix}; \quad Q^R_{\mathrm{rr}_1 2} = \begin{pmatrix} qr_{1,4} & qr_{1,5} & qr_{1,6} \\ qr_{2,4} & qr_{2,5} & qr_{2,6} \\ qr_{3,4} & qr_{3,5} & qr_{3,6} \end{pmatrix} \tag{65a}$$

$$Q^R_{\mathrm{rr}_2 1} = \begin{pmatrix} qr_{4,1} & qr_{4,2} & qr_{4,3} \\ qr_{5,1} & qr_{5,2} & qr_{5,3} \\ qr_{6,1} & qr_{6,2} & qr_{6,3} \end{pmatrix}; \quad Q^R_{\mathrm{rr}_2 2} = \begin{pmatrix} qr_{4,4} & qr_{4,5} & qr_{4,6} \\ qr_{5,4} & qr_{5,5} & qr_{5,6} \\ qr_{6,4} & qr_{6,5} & qr_{6,6} \end{pmatrix} \tag{65b}$$

$$Q^R_{\mathrm{rc}_1 1} = \begin{pmatrix} qr_{1,7} & qr_{1,8} & qr_{1,9} & qr_{1,10} & qr_{1,11} & qr_{1,12} & qr_{1,13} & qr_{1,14} & qr_{1,15} & qr_{1,16} \\ qr_{2,7} & qr_{2,8} & qr_{2,9} & qr_{2,10} & qr_{2,11} & qr_{2,12} & qr_{2,13} & qr_{2,14} & qr_{2,15} & qr_{2,16} \\ qr_{3,7} & qr_{3,8} & qr_{3,9} & qr_{3,10} & qr_{3,11} & qr_{3,12} & qr_{3,13} & qr_{3,14} & qr_{3,15} & qr_{3,16} \end{pmatrix} \tag{65c}$$

$$Q^R_{\mathrm{rc}_2 1} = \begin{pmatrix} qr_{4,7} & qr_{4,8} & qr_{4,9} & qr_{4,10} & qr_{4,11} & qr_{4,12} & qr_{4,13} & qr_{4,14} & qr_{4,15} & qr_{4,16} \\ qr_{5,7} & qr_{5,8} & qr_{5,9} & qr_{5,10} & qr_{5,11} & qr_{5,12} & qr_{5,13} & qr_{5,14} & qr_{5,15} & qr_{5,16} \\ qr_{6,7} & qr_{6,8} & qr_{6,9} & qr_{6,10} & qr_{6,11} & qr_{6,12} & qr_{6,13} & qr_{6,14} & qr_{6,15} & qr_{6,16} \end{pmatrix} \tag{65d}$$

By identification of the imaginary parts of aerodynamic forces Q^I_{rr} and Q^I_{rc} in Eqs. (63) and (64), we can write

$$Q^I_{\mathrm{rr}} = \begin{pmatrix} F \cdot C11 \cdot P & F \cdot C13 \cdot R \\ F \cdot C21 \cdot P & F \cdot C23 \cdot R \end{pmatrix} = \begin{pmatrix} Q^I_{\mathrm{rr}_1 1} & Q^I_{\mathrm{rr}_1 2} \\ Q^I_{\mathrm{rr}_2 1} & Q^I_{\mathrm{rr}_2 2} \end{pmatrix} \qquad Q^I_{\mathrm{rc}} = \begin{bmatrix} 0 & 0 \end{bmatrix}^T \tag{66}$$

The elements of the imaginary aerodynamic forces $Q^I_{\mathrm{rr}_1 1}$, $Q^I_{\mathrm{rr}_1 2}$, $Q^I_{\mathrm{rr}_2 1}$, $Q^I_{\mathrm{rr}_2 2}$, $Q^I_{\mathrm{rc}_1 1}$ and $Q^I_{\mathrm{rc}_2 1}$ are defined as follows

$$Q^I_{\mathrm{rr}_1 1} = \begin{pmatrix} qi_{1,1} & qi_{1,2} & qi_{1,3} \\ qi_{2,1} & qi_{2,2} & qi_{2,3} \\ qi_{3,1} & qi_{3,2} & qi_{3,3} \end{pmatrix} \quad Q^I_{\mathrm{rr}_1 2} = \begin{pmatrix} qi_{1,4} & qi_{1,5} & qi_{1,6} \\ qi_{2,4} & qi_{2,5} & qi_{2,6} \\ qi_{3,4} & qi_{3,5} & qi_{3,6} \end{pmatrix} \tag{67a}$$

$$Q^I_{\mathrm{rr}_2 1} = \begin{pmatrix} qi_{4,1} & qi_{4,2} & qi_{4,3} \\ qi_{5,1} & qi_{5,2} & qi_{5,3} \\ qi_{6,1} & qi_{6,2} & qi_{6,3} \end{pmatrix} \quad Q^I_{\mathrm{rr}_2 2} = \begin{pmatrix} qi_{4,4} & qi_{4,5} & qi_{4,6} \\ qi_{5,4} & qi_{5,5} & qi_{5,6} \\ qi_{6,4} & qi_{6,5} & qi_{6,6} \end{pmatrix} \tag{67b}$$

We next show the calculation of one single term corresponding to the real parts of the aerodynamic forces, because the same theory is used to calculate all of the terms of the real and imaginary aerodynamic forces. Please note that the F matrix is given by Eq. (51) and the $C12$ matrix is analytically given.

$$Q^R_{\mathrm{rr}_1 1} = F \cdot C12 = \begin{pmatrix} \cos\theta_0 & 0 & \sin\theta_0 \\ 0 & 1 & 0 \\ -\sin\theta_0 & 0 & \cos\theta_0 \end{pmatrix} \begin{pmatrix} c_{1,4} & c_{1,5} & c_{1,6} \\ c_{2,4} & c_{2,5} & c_{2,6} \\ c_{3,4} & c_{3,5} & c_{3,6} \end{pmatrix} = \begin{pmatrix} qr_{1,1} & qr_{1,2} & qr_{1,3} \\ qr_{2,1} & qr_{2,2} & qr_{2,3} \\ qr_{3,1} & qr_{3,2} & qr_{3,3} \end{pmatrix} \tag{68}$$

The first two columns of the $C12$ matrix are equal to zero (the first two columns of $C12$ are zero), whereas the coefficients in the last column are not equal to zero. By identification of the $Q^R_{\mathrm{rr}_1 1}$ matrix elements expressed by Eq. (68) and of the C matrix values given by Eq. (39) and other equations (not shown, due to the page number limitation):

$$\begin{aligned} & qr_{1,1} = 0; \qquad qr_{1,2} = 0; \\ & qr_{1,3} = c_{1,6} \cdot \cos\theta_0 + c_{3,6} \cdot \sin\theta_0 = \bar{S}[-a_{4,5} \cdot \cos(\theta_0 - \alpha_0) \\ & \quad - a_{5,5} \cdot \sin(\theta_0 - \alpha_0)] \\ & qr_{2,1} = 0; \qquad qr_{2,2} = 0; \qquad qr_{2,3} = c_{2,6} = \bar{S} \cdot a_{6,5} \\ & qr_{3,1} = 0; \qquad qr_{3,2} = 0; \\ & qr_{3,3} = -c_{1,6} \cdot \sin\theta_0 + c_{3,6} \cdot \cos\theta_0 = \bar{S}[a_{4,5} \cdot \sin(\theta_0 - \alpha_0) \\ & \quad - a_{5,5} \cdot \cos(\theta_0 - \alpha_0)] \end{aligned} \tag{69}$$

The coefficient a_i terms in Eq. (69) contain stability and control derivatives as seen in Eqs. (1–6). Therefore, we obtain, for all rigid aerodynamic elements Q^R, Table 1, in which the first three elements in column four were given in Eq. (69).

The first simulation scheme (Fig. 1) represents a system that uses the stability and control derivatives in the wind system of coordinates and in which the forces and moments are calculated in the aircraft system of coordinates. The second simulation scheme (Fig. 8) represents an equivalent scheme similar to the first, in which the states in the inertial system of coordinates are used in the closed loop.

V. Numerical Linearization Scheme

The formulations already developed in Secs. II, III, and IV will be validated for particular cases where the same types of stability and control derivatives are given in the wind system of coordinates. A problem appears if we add, subtract, or change a derivative or its initial value in the formulations, because then all the formulations change. Therefore, we need to develop automatic simulation formulations. We develop a MATLAB algorithm in which all of the preceding changes in coordinates and linearizations are automatically realized. We use the "dlinmod" MATLAB function which gives, in the state-space form, the linearized form of a system built in Simulink, around a specified trim point condition. The simulation scheme presented in Fig. 9 is equivalent to the scheme presented in Fig. 1 and takes into consideration the stability derivatives given in the wind coordinate system.

In block 1 of Fig. 9, the forces and moments F_w and M_w are calculated from the stability and control derivatives given in the wind system of coordinates. To simulate aircraft behavior, we convert

Table 1 Real part of Q_{rr} matrix (for displacements)

	x_i	y_i	z_i
X	0	0	$\bar{S} \cdot [-C_{d_h} \cdot \cos(\theta_0 - \alpha_0) - C_{\mathrm{Lift}_h} \cdot \sin(\theta_0 - \alpha_0)]$
Y	0	0	$\bar{S} \cdot C_{y_h}$
Z	0	0	$\bar{S} \cdot [C_{d_h} \cdot \sin(\theta_0 - \alpha_0) - C_{\mathrm{Lift}_h} \cdot \cos(\theta_0 - \alpha_0)]$
L	0	0	$\bar{S} \cdot b \cdot [C_{l_h} \cdot \cos(\theta_0 - \alpha_0) + C_{n_h} \cdot \sin(\theta_0 - \alpha_0)]$
M	0	0	$\bar{S} \cdot \bar{c} \cdot C_{m_h}$
N	0	0	$\bar{S} \cdot b \cdot [-C_{l_h} \cdot \sin(\theta_0 - \alpha_0) + C_{n_h} \cdot \cos(\theta_0 - \alpha_0)]$

these forces and moments, determined in block 1, to forces and moments in the aircraft coordinate system ΔF_a and ΔM_a, using block 2 for the transformation from the wind coordinate system to the aircraft coordinate system. The aircraft linear and angular speeds are the block 4 outputs and are calculated from the forces and moments in the aircraft coordinate system by use of the six degrees-of-freedom equations of motion. Parameters specific to the aircraft in the wind system of coordinates, such as the angle of attack, sideslip angle, and true airspeed, are further calculated in block 5 and used as inputs to block 1 in the aircraft time simulation. The scheme presented in Fig. 9 around the trim point specified in the simulation is then linearized by use of the dlinmod command in MATLAB, where the linear relationship between the inputs u and the outputs y is obtained under state-space form, in which the state vectors are $\mathbf{x} = (u, v, w, p, q, r, \phi, \theta, \psi, x_i, y_i, z_i)$. The components of the state vectors $\mathbf{x}$ are the linear speeds u, v, and w, the rates p, q, and r, the angles ϕ, θ, and ψ, and the three positions x_i, y_i, and z_i. The scheme presented in Fig. 9 can also be further represented under state-space form in Fig. 10.

Then, the state variables x are calculated with the first state-space system equations and are further used in the second equation of the state-space system to calculate the outputs y. In this case, we find the next scheme, equivalent to that shown in Fig. 10, in which the block 4 outputs, from the six degrees-of-freedom (DOF) equations of motion (see Fig. 9), are used.

As the schemes shown in Figs. 9 and 11 are compared, it is obvious that blocks 1, 2, and 4 are contained in the C and D matrices. The aircraft model thus obtained gives the variations of forces and moments calculated in the aircraft coordinate system dependent on the inputs u and state vectors $\mathbf{x}$. The matrices Q_{rr} and Q_{rc}, corresponding to rigid-to-rigid and to rigid-to-control mode interactions, respectively, are determined for aerodynamic forces in this way. The states multiplying these matrices, and the forces and moments, are calculated in the inertial system of coordinates. The scheme shown in Fig. 11 is redesigned by adding a first block, which changes the outputs y_i into y, and a second block, which changes the states x into the states x_i. This new scheme is presented in Fig. 12. Results obtained with analytical formulations are shown in Tables 1–4.

The changes to the coordinates implemented in the added blocks depend on the trigonometric functions of Euler angles, more specifically, the inputs are related to the outputs by nonlinear functions. The two blocks are then further linearized, which requires that the MATLAB command dlinmod be applied to the two blocks. For the block "x to x_i," a linear relationship is obtained:

$$x_i = D_1 x \tag{70}$$

which represents a simplified form of a state-space system where matrices A, B, and D_1 are zeros, as this block has no states. The linearization of block "y_i to y" gives

$$y = D_2 y_i \tag{71}$$

By using the blocks shown in Fig. 12, and Eqs. (70) and (71), we write

$$y = D_2 y_i = D_2(C_i x_i + D_i u) = D_2(C_i D_1 x + D_i u) = D_2 C_i D_1 x + D_2 D_i u \tag{72}$$

Identifying the matrices given by Eq. (72) with those given by the state-space equations

$$C = D_2 C_i D_1; \qquad D = D_2 D_i \tag{73}$$

The C and D matrices are obtained from the linearization presented in Fig. 9, and the D_1 and D_2 matrices are obtained by the linearization of two blocks shown in Fig. 12. We calculate the C_i and D_i matrices with Eqs. (73):

$$C_i = D_2^{-1} C D_1^{-1}; \qquad D_i = D_2^{-1} D \tag{74}$$

and the state vector $\mathbf{x}_i$ is:

$$\mathbf{x}_i = (x_i, y_i, z_i, \phi, \theta, \psi, \dot{x}_i, \dot{y}_i, \dot{z}_i, \dot{\phi}, \dot{\theta}, \dot{\psi})^T = [\,\eta_r \quad \dot{\eta}_r\,]^T \tag{75}$$

We can write the aerodynamic force equations for the rigid modes in the following form, similar to Eq. (60):

$$q_{\mathrm{dyn}}\left[Q_{\mathrm{rr}}^R \eta_r + \frac{\bar{c}}{2Vk} Q_{\mathrm{rr}}^I \dot{\eta}_r\right] = \left(q_{\mathrm{dyn}} Q_{rr}^R \quad q_{\mathrm{dyn}} \tfrac{\bar{c}}{2Vk} Q_{\mathrm{rr}}^I\right) x_i = C_i x_i \tag{76}$$

The real and imaginary parts of the aerodynamic forces corresponding to the rigid modes are obtained by identification from Eq. (76):

$$Q_{\mathrm{rr}}^R = \frac{1}{q_{\mathrm{dyn}}} C_i(1{:}6, 1{:}6); \qquad Q_{\mathrm{rr}}^I = \frac{2Vk}{\bar{c} q_{\mathrm{dyn}}} C_i(1{:}6, 7{:}12) \tag{77}$$

The control vector u is $\mathbf{u} = [\,\eta_c \quad 0\,]^T$. The real and imaginary parts of the aerodynamic forces corresponding to the interactions of rigid modes with the control modes are determined with the following equations:

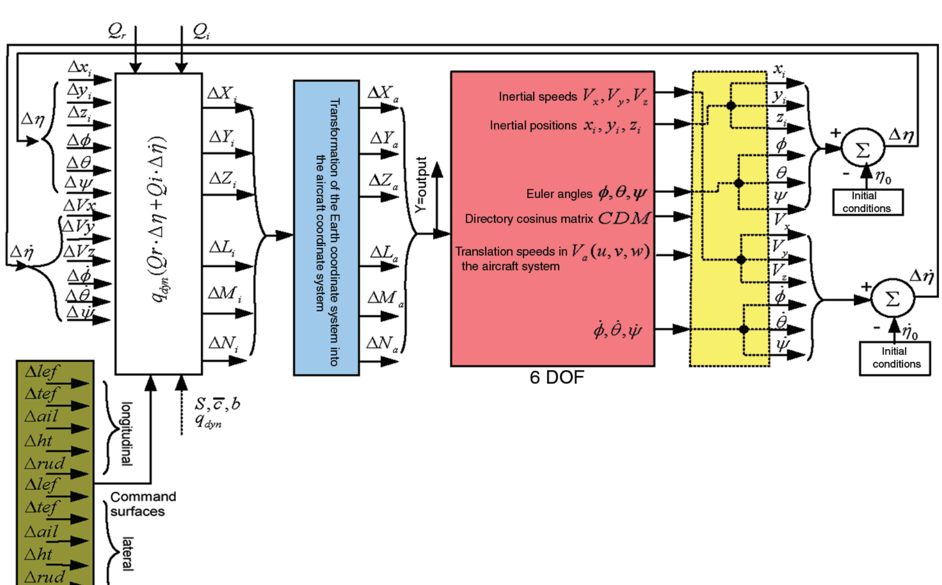

Fig. 8 Scheme for aerodynamic forces calculations from generalized coordinates.

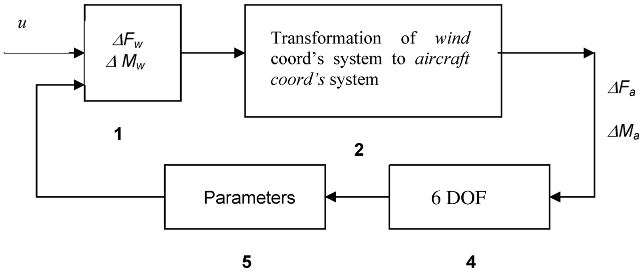

Fig. 9 Simulation aircraft scheme with stability and control derivatives given in the wind coordinates system.

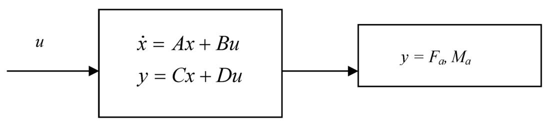

Fig. 10 Equivalence with the scheme shown in Fig. 9.

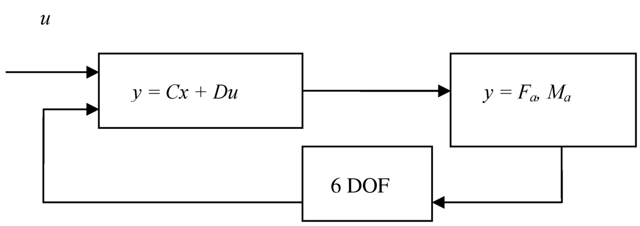

Fig. 11 Equivalence with the scheme shown in Fig. 2.

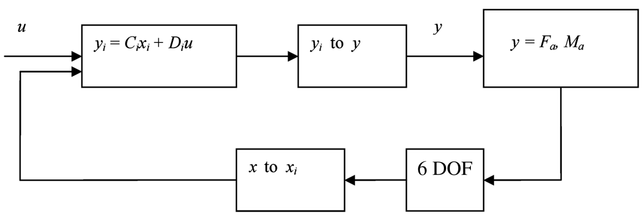

Fig. 12 Simulation scheme with forces and moments calculated in the inertial system of coordinates.

$$Q^R_{\mathrm{rc}} = \frac{1}{q_{\mathrm{dyn}}} D_i; \qquad Q^I_{\mathrm{rc}} = 0 \tag{78}$$

The algorithm is presented in the following three steps:

1) We apply the MATLAB command dlinmod to each of the three blocks to obtain C, D, D_1, and D_2 matrices.

2) Equations (77) and (78) are used to calculate the matrices Q_{rr} and Q_{rc}.

3) The initial matrices calculated with the doublet lattice method (DLM) or with the constant pressure method (CPM) calculated by finite element software, are replaced with the matrices obtained with our algorithm.

For validation purposes, the algorithm represents the numerical implementation (shown in Sec. V) of analytical implementation

Table 2 Real part of Q_{rr} and Q_{rc} matrix (for angles)

	ϕ	θ	ψ	δ
X	0	$-\bar{S}\cdot C_{d_\alpha}$	0	$-\bar{S}\cdot C_{d_\delta}$
Y	$\bar{S}\cdot C_{y_\beta}\cdot\sin\theta_0$	0	$-\bar{S}\cdot C_{y_\beta}$	$\bar{S}\cdot C_{y_\delta}$
Z	0	$-\bar{S}\cdot(-C_{\mathrm{Lift}_\theta}-C_{\mathrm{Lift}_\alpha})$	0	$-\bar{S}\cdot C_{\mathrm{Lift}_\delta}$
L	$\bar{S}\cdot b\cdot C_{l_\beta}\cdot\sin\theta_0$	0	$-\bar{S}\cdot b\cdot C_{l_\beta}$	$\bar{S}\cdot b\cdot C_{l_\delta}$
M	0	$\bar{S}\cdot\bar{c}\cdot(C_{m_\theta}+C_{m_\alpha})$	0	$\bar{S}\cdot\bar{c}\cdot C_{m_\delta}$
N	$\bar{S}\cdot b\cdot C_{n_\beta}\cdot\sin\theta_0$	0	$-\bar{S}\cdot b\cdot C_{n_\beta}$	$\bar{S}\cdot b\cdot C_{n_\delta}$

Table 3 Imaginary part of Q_{rr} matrix (for displacements)

	$\dot{x}_i$	$\dot{y}_i$	$\dot{z}_i$
X	$-\bar{S}\cdot C_{d_V}$	0	$-\bar{S}\cdot(C_{d_\alpha}/V_{x0})$
Y	0	$\bar{S}\cdot(C_{y_\beta}/V_{x0})$	0
Z	$-\bar{S}\cdot C_{\mathrm{Lift}_V}$	0	$-\bar{S}\cdot(C_{\mathrm{Lift}_\alpha}/V_{x0})$
L	0	$\bar{S}\cdot b\cdot C_{l_\beta}/V_{x0}$	0
M	$\bar{S}\cdot\bar{c}\cdot C_{m_V}$	0	$\bar{S}\cdot\bar{c}\cdot C_{m_\alpha}/V_{x0}$
N	0	$\bar{S}\cdot b\cdot C_{n_\beta}/V_{x0}$	0

Table 4 Imaginary part of Q_{rr} and Q_{rc} matrix (for angles)

	$\dot{\phi}$	$\dot{\theta}$	$\dot{\psi}$	$\dot{\delta}$
X	$-\bar{S}\cdot C_{d_p}$	$-\bar{S}\cdot C_{d_q}$	$\bar{S}\cdot\sin\theta_0\cdot C_{d_p}$	0
Y	$\bar{S}\cdot C_{y_p}$	0	$\bar{S}\cdot(-C_{y_p}\cdot\sin\theta_0+C_{y_r}\cdot\cos\theta_0)$	0
Z	$-\bar{S}\cdot C_{\mathrm{lft}_p}$	$-\bar{S}\cdot C_{\mathrm{Lift}_q}$	$\bar{S}\cdot\sin\theta_0\cdot C_{\mathrm{Lift}_p}$	0
L	$\bar{S}\cdot b\cdot C_{l_p}$	0	$\bar{S}\cdot b\cdot(-C_{l_p}\cdot\sin\theta_0+C_{l_r}\cdot\cos\theta_0)$	0
M	$\bar{S}\cdot\bar{c}\cdot C_{m_p}$	$\bar{S}\cdot\bar{c}\cdot C_{m_q}$	$-\bar{S}\cdot\bar{c}\cdot C_{m_p}\cdot\sin\theta_0$	0
N	$\bar{S}\cdot b\cdot C_{n_p}$	0	$\bar{S}\cdot b\cdot(-C_{n_p}\cdot\sin\theta_0+C_{n_r}\cdot\cos\theta_0)$	0

Table 5 Real part of Q_{rr} matrix obtained by analytical linearization

	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$
x	0	0	0	0	−614.96	0
y	0	0	0	−81.33	0	468.36
z	0	0	0	0	−1502.2	0
$\dot{x}$	0	0	0	−158.72	0	914.06
$\dot{y}$	0	0	0	0	−731.29	0
$\dot{z}$	0	0	0	367.85	0	−2118.3

Table 6 Real part of Q_{rr} matrix obtained by numerical linearization

	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$
x	0	0	−7.45E − 26	0	−614.96	0
y	0	0	0	−81.33	0	468.36
z	0	0	−4.38E − 25	0	−1502.2	0
$\dot{x}$	0	0	0	−158.72	0	914.06
$\dot{y}$	0	0	−6.16E − 23	0	−731.29	0
$\dot{z}$	0	0	0	367.85	0	−2118.3

Table 7 Imaginary part of Q_{rr} matrix obtained by analytical linearization

	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$
x	−0.04	0	−0.66	0.08	−8.6	−0.013892
y	0	−0.50	0	2.16	0	99.091
z	−0.08	0	−1.61	0.08	−800.16	−0.013892
$\dot{x}$	0	−0.98	0	−5412.5	0	3948.3
$\dot{y}$	−0.92	0	−0.78	1.3824	−15, 977	−0.24005
$\dot{z}$	0	2.27	0	−704.62	0	−2809.5

Table 8 Imaginary part of Q_{rr} matrix obtained by numerical linearization

	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$
x	−0.04	0	−0.66	0.08	−8.6	−0.013892
y	0	−0.50	0	2.16	0	99.091
z	−0.08	0	−1.61	0.08	−800.16	−0.013892
$\dot{x}$	0	−0.98	0	−5412.5	0	3948.3
$\dot{y}$	−0.92	0	−0.78	1.3824	−15, 977	−0.24005
$\dot{z}$	0	2.27	0	−704.62	0	−2809.5

presented in Secs. II, III, and IV. A comparison between the obtained values with the analytical formulation and the numerical formulation presented here is given in Tables 5–8.

We can see that the obtained results are the same. The numerical algorithm allowed us to obtain the same results for 90 flight test conditions as the ones obtained analytically.

VI. Conclusions

In this paper, we used two approaches, analytical (Secs. II, III, and IV) and numerical (Sec. V), to validate the aerodynamic force formulations corresponding to rigid-to-rigid and rigid-to-control interaction modes for aeroservoelasticity studies: only from the knowledge of the stability and control derivatives in the wind system of coordinates. These derivatives were obtained at NASA DFRC for 90 flight test conditions for an F/A-18 aircraft represented by Mach numbers, altitudes, and angles of attack.

The formulations (numerical and analytical) presented here can be applied on another aircraft and, therefore, may use a different set of stability and control derivatives. The numerical approach will likely be much faster than the analytical one, due to the long time taken by successive linearizations. The analytical approach may become more useful in the future.

This approach, which was developed in Secs. II, III, and IV, allows us to obtain the analytical formulas for all the stability and control derivatives in the inertial system from those calculated in the wind system of coordinates. Linearizations at the trim condition are performed at each calculation step.

The second approach consists of a numerical linearization of the simulation scheme in the wind reference system of coordinates. Values of stability and control derivatives obtained with this method and those calculated analytically with the first method are the same.

Values of aerodynamic forces corresponding to rigid and control modes are validated by use of both approaches (numerical and analytical) for 90 flight test conditions.

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